Honors Calculus Test 2023 University of Delaware Honors Program

This test is to be taken **only** by students who wish to enroll in the Honors section of Math 242 (Calculus II). Other students will be placed in regular sections of the math courses they qualify for, determined by their scores on the AP Calculus AB or BC Exam or the UD Math Placement Test.

The solutions are due before June 26, 2023. Please scan your solution, check it for legibility, and email it to askhonors@udel.edu with the subject line Honors Calculus Test, with your UDel username and UDel ID in the message.

- You may wish to consult books for the problems.
- For correct placement, please do not consult anyone when answering the questions.
- The problems are challenging and just partial solutions of some of the problems are needed to pass the test. Please submit an organized version of **whatever you have managed to do.**
- Please work on the problems over several days, if needed. These are non-standard problems and require **perseverance** and skill to solve them.
- Please show all your work that is more important than the final answer.
- Please organize your work and write **neatly**. Use extra sheets of paper if needed.
- Please write to the Honors office in case you need more time for the test

Name -

UD ID Number -

Circle the calculus proficiency attained including the score, if you know it.

AP-AB Exam:-

AP-BC Exam:- Other:-

1. A cone of radius 5 cms and height 10 cms is lowered, vertex first, into water in a cylindrical tank of radius 20 cms. The vertex of the cone approaches the bottom of the tank at a speed of 1 cm per second. How fast is the water level rising in the tank when half the height of the cone is submerged?

Please define the variables before you use them. We do not want to guess what the variables represent.



2. The figure shows a curve C with the property that, for every point P(a, b) on the middle curve $y = x^2$, the areas of the regions L and M are equal. Find the equation of the curve C.



3. Find **ALL** positive functions f(x) such that

$$\left(\int f(x) \, dx\right) \left(\int \frac{1}{f(x)} \, dx\right) = -1.$$

The goal is to find all the solutions. If you are unable to find all the solutions, give the solutions you have found and show your work where you verified that these are solutions of the equation.