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1 INTRODUCTION

In essence, the boundary element method (BEM) may be considered as an application of the finite element method (FEM), designed originally for the numerical solutions of partial differential equations (PDE) in the domains, to the boundary integral equations (BIE) on closed boundary manifolds. The terminology of BEM originated from the practice of discretizing the boundary manifold of the solution domain for the BIE into boundary elements, resembling the term of finite elements in FEM. As in FEM, the use of the terminology *boundary element* in the literature is in two different contexts: the boundary manifolds are decomposed into boundary elements, which are geometric objects, while the boundary elements for approximating solutions of BIEs are actually the finite element functions defined on the boundaries. Looking through the literature, it is difficult to trace back one fundamental research paper and the individuals who were responsible for the historical development of the BEM. However, from the computational point of view, the work by Hess and Smith deserves mention as one of the cornerstones of BEMs. In their 1966 paper (Hess and Smith, 1966), boundary elements (or rather, surface elements) have been used to approximate various types of bodies and to calculate the potential flow about arbitrary bodies. On the other hand, the paper by Nedelec and Planchard (1973) may be considered as a genuine work on boundary element paper with respect to the variational formulation of BIEs. Other early contributions to the development of boundary element in the 1960s and 1970s from the mathematical point of view include (Fichera, 1961; Wendland, 1965, 1968; MacCamy, 1966; Mikhlin, 1970; Hsiao and MacCamy, 1973; Stephan and Wendland, 1976; Jaswon and Symm, 1977; LeRoux, 1977; Nedelec, 1977, and Hsiao and Wendland, 1977), to name a few.

The BEM has received much attention and gained wide acceptance in recent years. From 1989 to 1995, the German Research Foundation, DFG, installed a Priority Research Program "Boundary Element Methods", and the final report appeared as a book (Wendland, 1997). There has been an increasing effort in the development of efficient finite element solutions of BIEs arising from elliptic boundary

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