

# Applications of Integral Equation Methods to a Class of Fundamental Problems in Mechanics and Mathematical Physics

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## Abstract

The monograph *Singular Integral Equations* by N.I. Muskhelishvili was published originally in Russian in 1946 and was revised and translated into English in 1958. In this monograph, the solution of the Dirichlet problem is expressed in terms of the potential of a simple layer, which leads to a Fredholm integral equation of the first kind. This new approach introduced by Muskhelishvili in 1946 for solving boundary value problems by using integral equations of the first kind has made significant contribution 30 years later to the development of variational methods for boundary integral equations and their numerical discretizations. The later is known as the boundary element method and has become one of the most popular numerical schemes in nowadays.

To pay a high tribute to Muskhelishvili in celebrating his 120th birthday anniversary, this lecture discusses boundary integral equations of the first kind and its applications to a class of fundamental problems in elasticity, fluid mechanics and other branches of mathematical physics. Applications are drawn from various disciplines including topics such as singular perturbation theory for viscous flow past an obstacle, boundary variational inequalities for contact problems in elasticity, coupling procedure and domain decomposition for interface problems in non homogeneous medium. The presentation of these topics indicates also the chronological order of the development of the Muskhelishvili's method concerning first kind integral equations and its generalizations.