



Dean Tom Apple
and the College of Arts and Sciences
cordially invite you to
*the Inaugural Lecture of George C. Hsiao as
Carl J. Rees Professor in Mathematics*

"Stokes Paradox and Its Consequences"

4:00p.m., Tuesday, November 29, 2005
004 Kirkbride

Reception immediately following the lecture
Blue and Gold Club, 44 Kent Way

*Please respond by November 22 to (302)851-2793
or rsp@art-sci.udel.edu*

Born in Shanghai, China, Dr. Hsiao is an engineering graduate of the National Taiwan University. He received his master's degree in civil engineering from Carnegie Institute of Technology and his doctorate in Mathematics from Carnegie-Mellon University. In 1969, Dr. Hsiao joined the Department of Mathematical Sciences at the University of Delaware, where he has been a full professor since 1977. He was named the Carl J. Rees Chair Professor in Mathematics at the University in 2005.

Dr. Hsiao's education in dual disciplines has influenced his research interests, which include integral equations, partial differential equations, singular perturbation theory, elasticity and fluid dynamics, wavelets, and direct and inverse problems in acoustic and electromagnetic scattering. He is one of the leading experts and authorities on variational and boundary element methods for integral equations. The author of more than 150 papers on mathematics, applied mechanics, oceanic environment, rheology and biomedical engineering, Dr. Hsiao has given invited lectures all over the world. He is the co-author of *Maple Projects for Differential Equations; Water Waves and Ship Hydrodynamics: An Introduction*, and *Boundary-field Equation Methods for a Class of Nonlinear Problems*.

Dr. Hsiao was an Alexander von Humboldt-Stiftung senior Fellow and has been awarded three time research Humboldt fellowships in Germany. Recognized for his excellence as an educator, Dr. Hsiao was the recipient of the 1996 College of Arts and Science Outstanding Teacher Award and the 2000 winner of the Francis Alison Medal, the highest faculty honor at the University of Delaware, in recognition of his scholarship, professional achievement and dedication.



DEPARTMENT OF MATHEMATICAL SCIENCES

CARL J. REES PROFESSOR INAUGURAL LECTURE

Dr. George C. Hsiao
Department of Mathematics
University of Delaware

Stokes Paradox and its Consequences

Tuesday, November 29, 2005
Kirkbride 004
4:00-5:00

In this lecture, I would like to share my research experience of 30 years in applied mathematics with the audience. I began my research by solving a simple boundary-value problem for the Stokes equation. Over the years, this problem has taken me in many unexpected directions, and this journey has proved to be an extremely rewarding experience.

Carl J. Rees Professor Inaugural Lecture

November 29, 2005

Stokes Paradox and its Consequences

George C. Hsiao

www.math.udel.edu/~hsiao

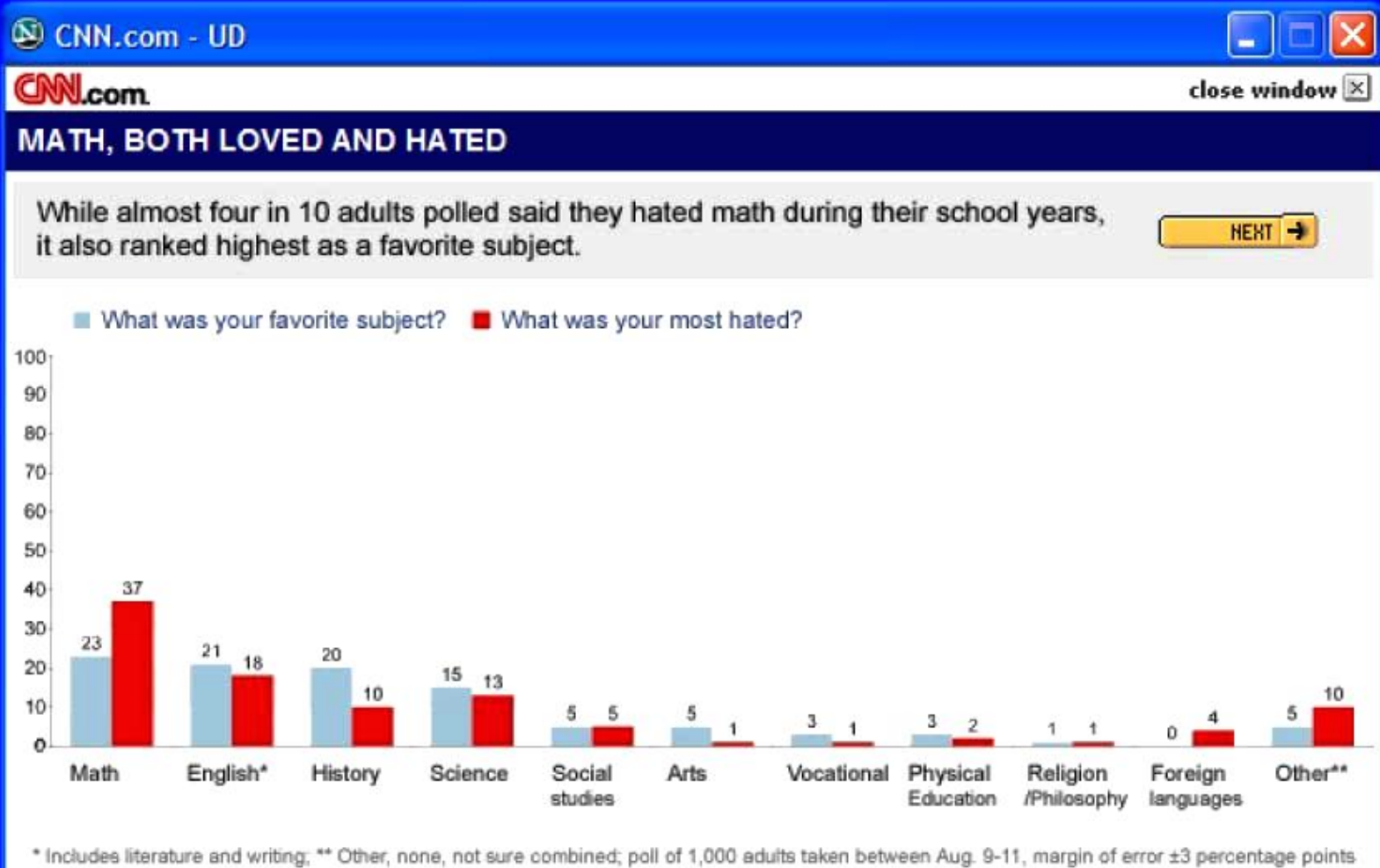
Department of Mathematical Sciences

UNIVERSITY OF DELAWARE

Prologue



CNN-Poll



The Most Notorious Statement in the History of Mathematics

**I have discovered
a truly marvelous
proof of this
which, however,
this margin is too
narrow to hold.**

- Fermat in Paris, 1621



*From: Newsletter Vol. 2, No.1, 1994, Dept. of Math Sciences,
University of Delaware edited by T. S. Angell and Ivar Stakgold*

Fermat's Last Theorem

Fermat's Last Theorem (FLT) asserts that there are no positive **integers** X , Y , and Z such that

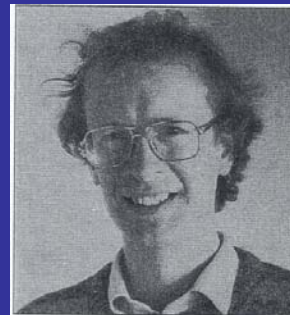
$$X^n + Y^n = Z^n$$

in which n is a natural number **greater than** 2.

- Andrew John Wiles proved Fermat's Last Theorem in 1994 (with the help from Richard Taylor). It had taken mathematicians only **329** years since Fermat's death in 1665 to settle the problem.
- **Theorem of Pythagoras:** $X^2 + Y^2 = Z^2$

Andrew John Wiles (1953 --)

- Andrew John Wiles: (born April 11, 1953) is a British mathematician living in the United States. In 1974, he received his bachelor's degree from the University of Oxford. He then completed his PhD. at the University of Cambridge in 1979 and is currently a Professor at Princeton University.
- Wiles received the 1995-1996 Wolf Prize for spectacular contributions to number theory and related fields, for major advances to number theory on **fundamental conjectures** and for Settling Fermat's Last Theorem.



Andrew J. Wiles

Outline

- Stokes Paradox
- Boundary Element Methods
- Some Numerical Experiments
- Ongoing Research: ψ dOs
- Concluding Remarks

Stokes Paradox

UNIVERSITY OF DELAWARE

Department of Chemical Engineering
Fluid Mechanics Seminar

Wednesday, April 22 1:30 P.M.

105 Colburn

Prof. George Hsiao
Mathematics Department
University of Delaware

"Stokes Paradox"

Professor Hsiao is an applied mathematician with a masters degree in civil engineering, specializing in two-phase (solid liquid) flow. His more recent interests, growing out of the earlier work, are concerned with singular perturbations of the Navier-Stokes equations. All members of chemical engineering with fluid mechanics interests will wish to get to know him. The seminar will start promptly at 1:30 since Prof. Hsiao has a 2:30 class.

1970 Fluid Mechanics Seminar in Chemical Engineering

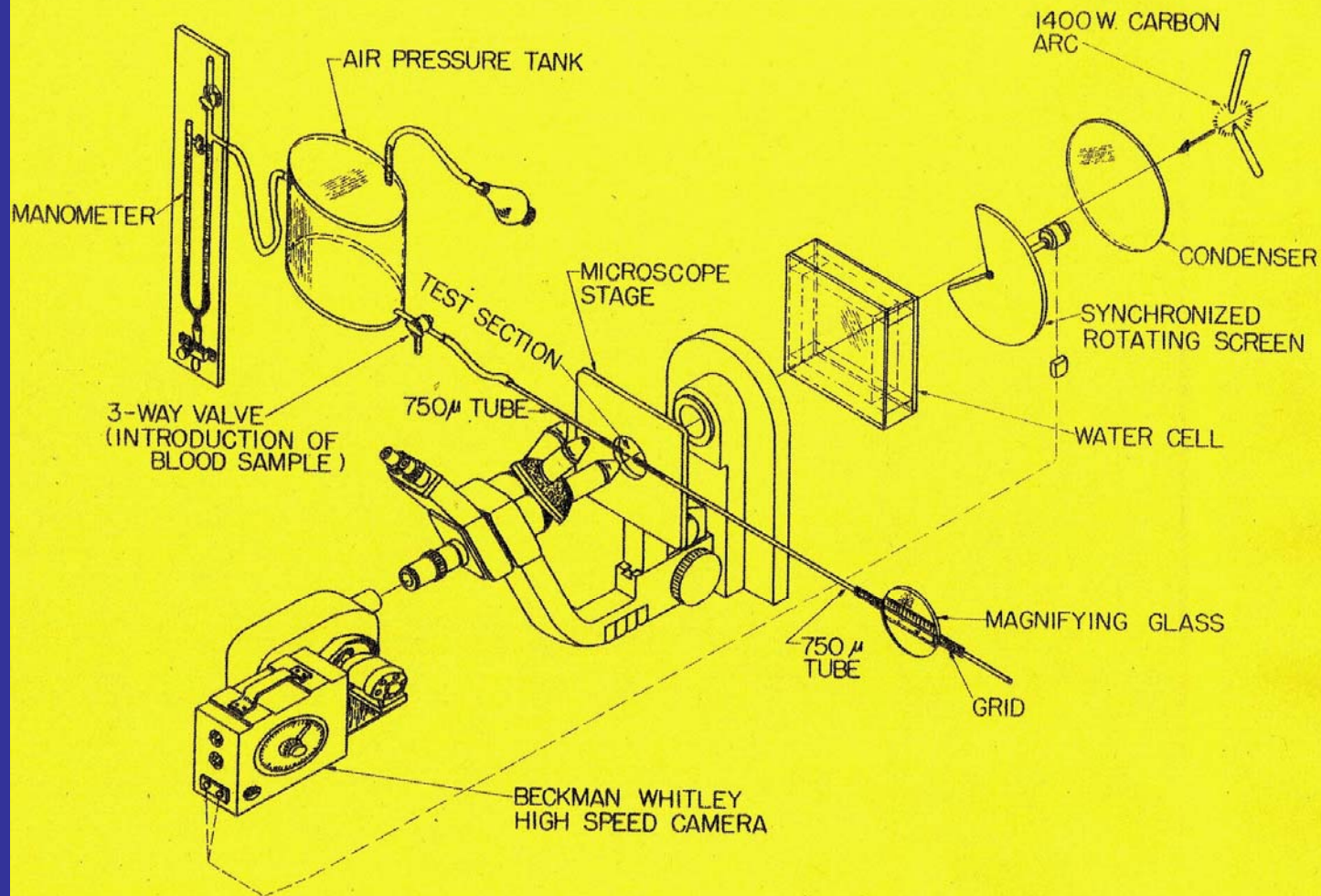
Department of Chemical Engineering
Fluid Mechanics Seminar
Wednesday, April 22 1:30 P.M.
105 Colburn

Prof. George Hsiao
Mathematics Department
University of Delaware

"Stokes Paradox"

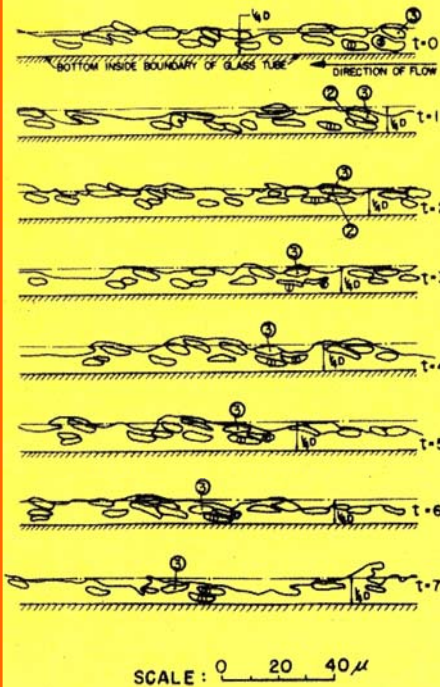
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Apparatus for Blood Flow



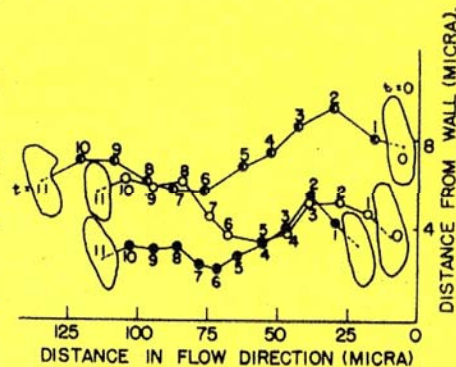
The Profile Viscosity and Other Characteristics of Blood Flow in a Non-uniform Shear Field

GEORGE BUGLIARELLO,* CHANDRA KAPUR, and GEORGE HSIAO, *Civil Engineering Department, Carnegie Institute of Technology, Pittsburgh, Pennsylvania*



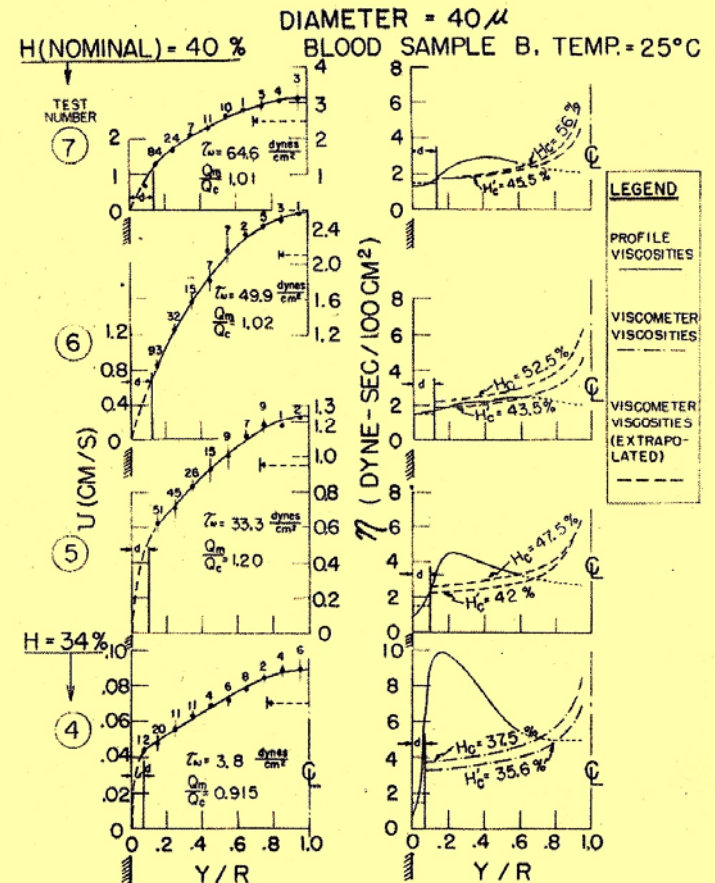
TEST NO.5-A

TUBE DIAMETER $D = 39.8 \mu$
 HEMATOCRIT $H = 29.5 \%$
 AVERAGE VELOCITY $v_M = 4.9 \text{ MM/Sec.}$
 1 TIME UNIT = 1.33 M Sec.

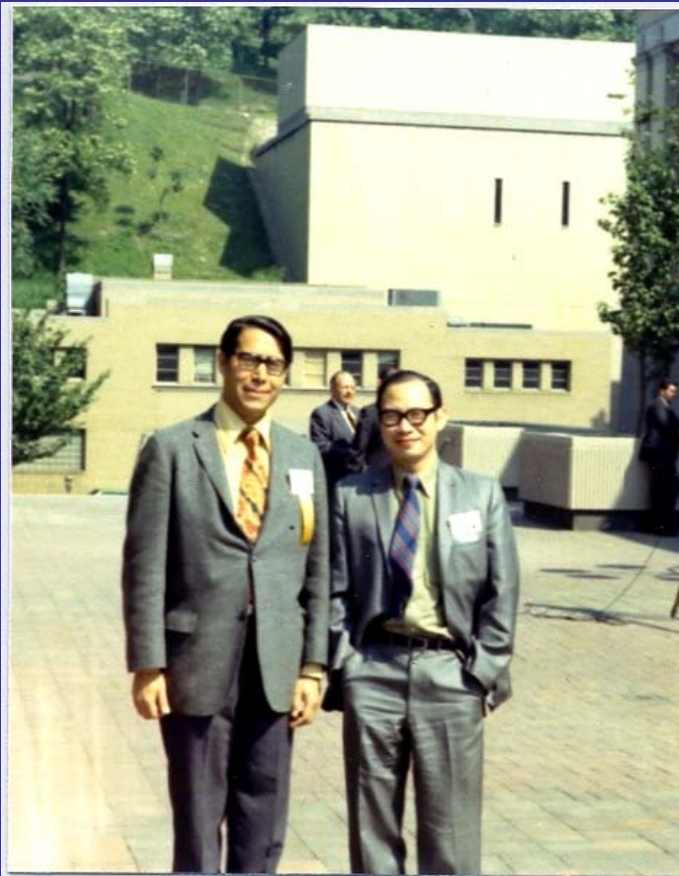


Velocity and viscosity profiles for representative tests at higher concentrations in the 40-m tube.

Blood Flow in Vitro



1974 ASCE Meeting



Pittsburgh, Pa.

- **George Bugliarello:**
University Professor,
Chancellor and
former President (1973-94)
of Polytechnic University in
Brooklyn, NY.
- **My mentor, collaborator
and dear friend.**

Metzner: Flow of Non-Newtonian Fluids

HANDBOOK OF FLUID DYNAMICS

VICTOR L. STREETER, Editor-in-Chief

*Professor of Hydraulics
University of Michigan*

FIRST EDITION

McGRAW-HILL BOOK COMPANY, INC.

NEW YORK TORONTO LONDON

1961

Section 7

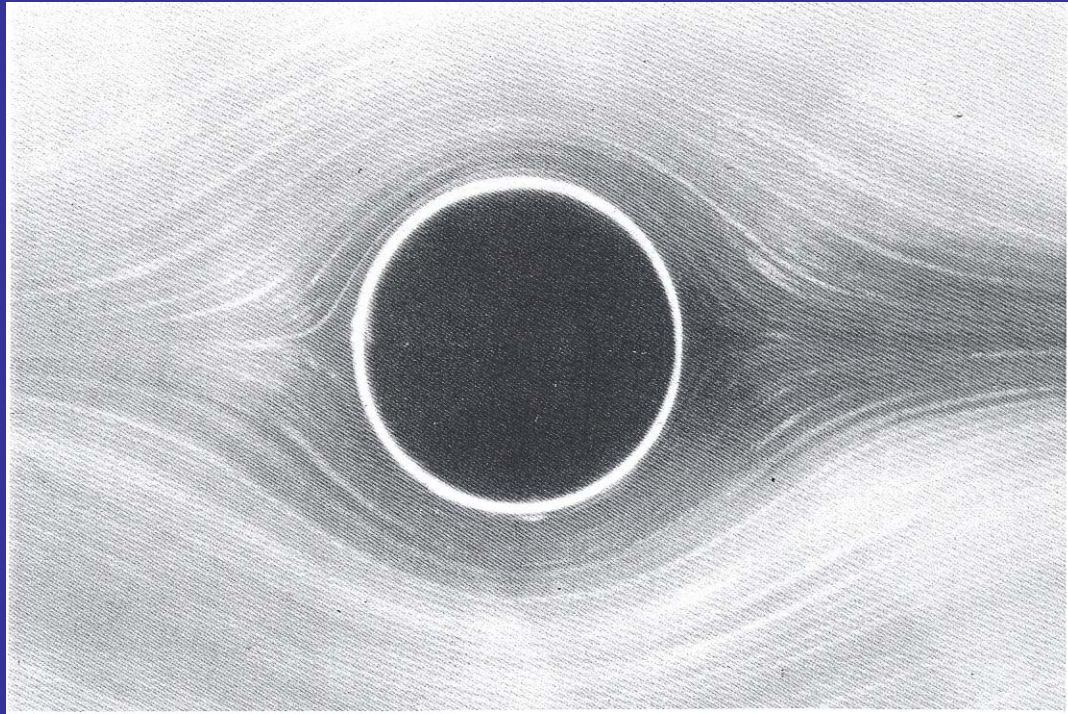
FLOW OF NON-NEWTONIAN FLUIDS

By

A. B. METZNER, *University of Delaware, Newark, Delaware*

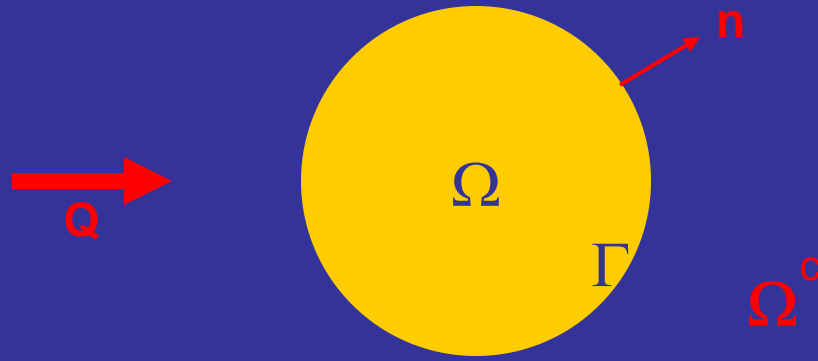
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Viscous Fluid Flow



Photograph by Sadatoshi Taneda in
“An Album of Fluid Motion”, assembled by Milton van Dyke

Viscous Fluid Flow



Boundary Value Problem (P_{Re})

(in dimensionless form) $\Omega^c := \mathbb{R}^2 \setminus \overline{\Omega}$

$$-\Delta \mathbf{u} + Re(\mathbf{u} \cdot \nabla \mathbf{u}) + \nabla p = \mathbf{0}; \quad \nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega^c,$$

$$\mathbf{u}|_{\Gamma} = \mathbf{0} \quad \text{on } \Gamma = \partial\Omega$$

$$\mathbf{u} \rightarrow \mathbf{u}_{\infty} := \mathbf{Q}/|\mathbf{Q}|, \quad p \rightarrow 0 \quad \text{as } |\mathbf{x}| \rightarrow \infty.$$

Reynolds number. $Re := \frac{D|\mathbf{Q}|}{\mu/\rho} \ll 1.$

Force. $\mathbf{F} = \int_{\Gamma} \underline{\underline{\sigma}}[\mathbf{n}] ds, \quad \underline{\underline{\sigma}} = -p\underline{\underline{I}} + (\nabla \mathbf{u} + \nabla \mathbf{u}^T).$

Boundary Value Problem (P_{Re})

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$$-\Delta \mathbf{u} + Re(\mathbf{u} \cdot \nabla \mathbf{u}) + \nabla p = \mathbf{0}; \quad \nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega^c,$$

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Reynolds number. $Re := \frac{D|\mathbf{Q}|}{\mu/\rho} \ll 1.$

Force. $\mathbf{F} = \int_{\Gamma} \underline{\underline{\sigma}}[\mathbf{n}] ds, \quad \underline{\underline{\sigma}} = -p\underline{\underline{I}} + (\nabla \mathbf{u} + \nabla \mathbf{u}^T).$

● **Stokes Paradox.** *The reduced problem (P_0) has no solution.*

Main Results

Theorem (Hsiao and MacCamy 1982).

- For any given constant vector \mathbf{A} , there exists a unique "solution" pair (\mathbf{u}, p) of the modified Stokes problem:

$$-\Delta \mathbf{u} + \nabla p = \mathbf{0}; \quad \nabla \cdot \mathbf{u} = 0 \quad \text{in} \quad \Omega^c,$$

$$\mathbf{u}|_{\Gamma} = \mathbf{0} \quad \text{on} \quad \Gamma,$$

$$\mathbf{u} - \mathbf{A} \log |\mathbf{x}| = O(1), \quad p = o(1) \quad \text{as} \quad |\mathbf{x}| \rightarrow \infty.$$

- The force \mathbf{F} admits the asymptotic expansion

$$\mathbf{F} = 4\pi \left\{ \frac{\mathbf{A}_1}{(\log Re)} + \frac{\mathbf{A}_2}{(\log Re)^2} \right\} + O((\log Re)^{-3})$$

as $Re \rightarrow 0^+$,

where the constant vectors \mathbf{A}_1 and \mathbf{A}_2 are coincided with those obtained by the singular perturbation procedure developed by Hsiao and MacCamy in [1973].

Moreover, the constant vector \mathbf{A}_1 is independent of the shape of the obstacle.

Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

942

Theory and Applications of Singular Perturbations

Proceedings, Oberwolfach 1981

Edited by W. Eckhaus and E.M. de Jager



Springer-Verlag
Berlin Heidelberg New York

SINGULAR PERTURBATIONS FOR THE TWO-DIMENSIONAL

VISCOUS FLOW PROBLEM

George C. Hsiao*
University of Delaware
Newark, Delaware 19711
U.S.A.

and Richard C. MacCamy**
Carnegie-Mellon University
Pittsburgh, Pennsylvania 15213
U.S.A.

ABSTRACT

This paper is concerned with the validity of the method of matched inner and outer expansions for treating the two-dimensional steady flow of a viscous, incompressible fluid past an arbitrary obstacle. In particular, it is shown that the force exerted on the obstacle by the fluid admits the asymptotic representation:

$$\mathbf{F} = 4\pi\{\mathbf{A}_1(\log R)^{-1} + \mathbf{A}_2(\log R)^{-2}\} + O((\log R)^{-3})$$

as the Reynolds number $R \rightarrow 0^+$, where \mathbf{A}_i 's are constant vectors which are the same as those obtained by the matching procedure formulated previously by the authors. This asymptotic representation formula agrees also, up to terms of $O((\log R)^{-2})$, with the expression from the solution of the complete Oseen boundary-value problem; in fact, it is seen that these calculations are as accurate as those from the Oseen solution, since the Oseen solution is no longer a valid approximation to the solution of the viscous flow problem for terms of order higher than $(\log R)^{-2}$. Proofs involve simple layer potentials and asymptotic estimates for solutions of various linearized Navier-Stokes equations.

1. Introduction

In recent years, there has been an increasing effort to establish the validity of formal expansions constructed by the method of matched asymptotic expansions for treating the problem of the two-dimensional viscous flow past a cylinder as indicated from a number of investigations on the Lagerstrom model as well as its variants [1,2,11,13,21,24], to name a few. The purpose of this paper is to show that the conclusions on the validity of formal expansions for the model problems in [11] and [13] remain also true for the viscous flow problem. That is, we shall show that the formal expansions based on the matching procedure previously formulated by Hsiao and MacCamy [12] for the Navier-Stokes

*The work of this author was supported in part by the Alexander von Humboldt Foundation, Germany.

**The work of this author was supported in part by the National Science Foundation under Grant MCS-800-1944.

Mathematical Ingredients

- Asymptotic Analysis/Singular Perturbation Theory
- Boundary Integral Equations (of first kind) /Boundary Element Methods

An Historical Paper

SIAM REVIEW

Vol. 15, No. 4, October 1973

SOLUTION OF BOUNDARY VALUE PROBLEMS BY INTEGRAL EQUATIONS OF THE FIRST KIND*

GEORGE HSIAO[†] AND R. C. MACCAMY[‡]

Abstract. This paper discusses an integral equation procedure for the solution of boundary value problems. The method derives from work of Fichera and differs from the more usual one by the use of integral equations of the first kind. The method here extends to equations of higher order than second. Its connection with singular perturbation theory and thin-body theory are indicated by examples. Some numerical experiments are included to indicate how the method operated in specific situations.

Bellman's Letter (February 1, 1974)

MATHEMATICS IN SCIENCE AND ENGINEERING

A Series of Monographs and Textbooks

Editor:

RICHARD BELLMAN
Departments of Mathematics and Engineering
University of Southern California
University Park
Los Angeles, California 90007

Publishers:

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New York, New York 10003

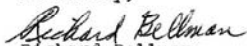
February 1, 1974

Dr. George C. Hsiao
Department of Mathematics
University of Delaware
Newark, Delaware 19711

Dear Dr. Hsiao:

I read your paper entitled "Singular Perturbations for a Nonlinear Differential Equation with a Small Parameter," with great interest. Have you given any thought to extending these results and collecting them in book form? If so, I would be glad to consider it for my Academic Press series.

Cordially,


Richard Bellman

RB:rk

cc: Mr. Ryo Arai

I read your paper entitled "Singular Perturbations for a Nonlinear Differential Equation with a small parameter," with great interest. Have you given any thought to extend these results and collecting them in book form? If so, I would be glad to consider it for my Academic Press series.

-Richard Bellman

TH Darmstadt (1975-76)

FACHBEREICH MATHEMATIK
DER TECHNISCHEN HOCHSCHULE DARMSTADT

61 Darmstadt, den 27.4.1976

E I N L A D U N G

z u m
Kolloquium über Mathematik

Am Donnerstag, dem 6. Mai 1976, um 17.15 Uhr, in Raum 11/23
wird:

Herr Professor G.C. Hsiao (Universität of Delaware, z.Zt. Darmstadt)
über das Thema

Boundary value problems for quasilinear elliptic equations
sprechen.

Tee: 16.30 Uhr, in Raum 2d/406

- - - - -

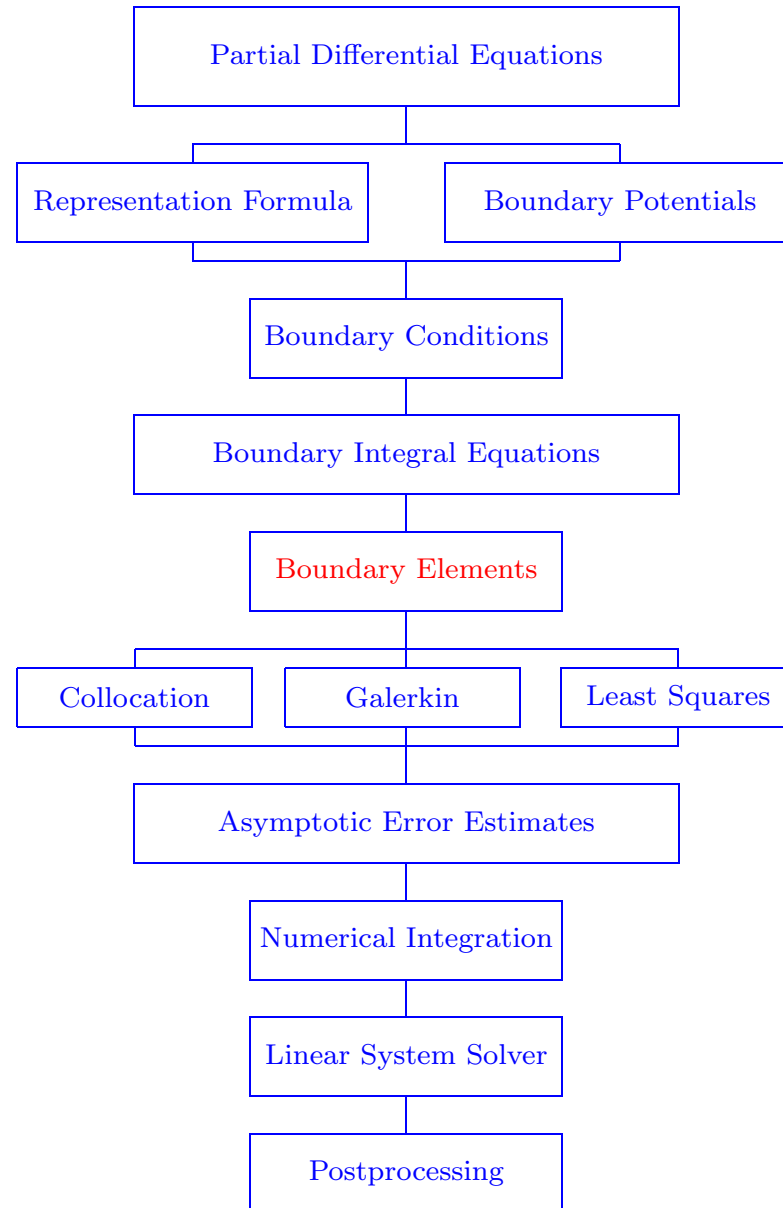
Der Vortrag von Herrn Hsiao



Boundary Element Methods

UNIVERSITY OF DELAWARE

A Schematic Procedure for BEM



Two Fundamental Papers

Reprinted from JOURNAL OF MATHEMATICAL ANALYSIS AND APPLICATIONS
All Rights Reserved by Academic Press, New York and London

Vol. 58, No. 3, May 1977
Printed in Belgium

A Finite Element Method for Some Integral Equations of the First Kind*

GEORGE C. HSIAO

University of Delaware, Newark, Delaware 19711

AND

WOLFGANG L. WENDLAND

Technische Hochschule Darmstadt, Darmstadt, Germany

Submitted by J. L. Lions

This paper discusses a finite element approximation for a class of singular integral equations of the first kind. These integral equations are deduced from Dirichlet problems for strongly elliptic differential equations in two independent variables. By a variation of technique due to Aubin, it is shown that the Galerkin method with finite elements as trial functions leads to an optimal rate of convergence.

Offprint from "Archive for Rational Mechanics and Analysis",
Volume 94, Number 2, 1986, pp. 179–192
© Springer-Verlag 1986
Printed in Germany

On the Stability of Integral Equations of the First Kind with Logarithmic Kernels

GEORGE C. HSIAO

Communicated by G. FICHERA

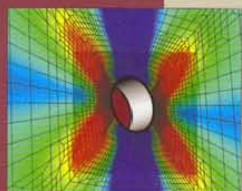
Abstract

This paper discusses the stability of the Galerkin method for a class of boundary integral equations of the first kind. These integral equations arise in acoustics, elasticity, and hydrodynamics, and the kernels of the principal parts of the corresponding integral operators all have logarithmic singularities. It is shown that an optimal choice of the mesh size can be made in the numerical computation so that one will obtain an optimal rate of convergence of the approximate solutions. The results here are consistent with those obtained by the Tikhonov regularization procedure.

Fundamental Concepts

- Weak solutions for the boundary integral equations and their intimate relations with those of the corresponding PDE's
- Stability of the boundary integral equations and its dependence on the **order** of the BIO's.

Editors: Erwin Stein | René de Borst | Thomas J.R. Hughes



Encyclopedia of Computational Mechanics

Volume 1 Fundamentals

Volume 2 Solids and Structures

Volume 3 Fluids



2004 John Wiley & Sons, Ltd

Chapter 12

Boundary Element Methods: Foundation and Error Analysis

G. C. Hsiao¹ and W. L. Wendland²

¹ University of Delaware, Newark, DE, USA

² Universität Stuttgart, Stuttgart, Germany

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1 INTRODUCTION

In essence, the boundary element method (BEM) may be considered as an application of finite element method (FEM), designed originally for the numerical solutions of partial differential equations (PDE) in the domains, to the boundary integral equations (BIE) on closed boundary manifolds. The terminology of BEM originated from the practice of discretizing the boundary manifold of the solution domain for the BIE into boundary elements, resembling the term of finite elements in FEM. As in FEM, in the literature, the use of the terminology *boundary element* is in two different contexts: the boundary manifolds are decomposed into boundary elements, which

are geometric objects, while the boundary elements for approximating solutions of BIEs are actually the finite element functions defined on the boundaries. Looking through the literature, it is difficult to trace back one fundamental research paper and the individuals who were responsible for the historical development of the BEM. However, from the computational point of view, the work by Hess and Smith deserves mention as one of the cornerstones of BEM. In their 1966 paper (Hess and Smith, 1966), boundary elements (or rather surface elements) have been used to approximate various types of bodies and to calculate the potential flow about arbitrary bodies. On the other hand, the paper by Nedelec and Planchard (1973) may be considered as a genuine boundary element paper with respect to the variational formulation of BIEs. Other early contributions to the boundary element development in the 1960s and 1970s from the mathematical point of view include Fichera (1961), Wendland (1965, 1968), MacCamy (1966), Mikhlin (1970), Hsiao and MacCamy (1973), Stephan and Wendland (1976), Jaswon and Symm (1977), LeRoux (1977), Nedelec (1977), and Hsiao and Wendland (1977), to name a few.

The BEM has received much attention and gained wide acceptance in recent years. From 1989 to 1995, the German Research Foundation DFG installed a Priority Research Program 'Boundary Element Methods', and the final report appeared as a book (see Wendland, 1997). There has been an increasing effort in the development of efficient finite element solutions of BIEs arising from elliptic boundary value problems (BVP). In fact, nowadays, the term BEM denotes any 'efficient method' for the approximate numerical solution of these boundary integral equations.

Encyclopedia of Computational Mechanics, Edited by Erwin Stein, René de Borst and Thomas J.R. Hughes, Volume 1: *Fundamentals*. © 2004 John Wiley & Sons, Ltd. ISBN: 0-470-84699-2.

A Pleasant Surprise

Professor David L. Russell is the former Ph.D. thesis adviser of the first author (G.C) at the University of Wisconsin-Madison. His work and personality have had a profound influence in shaping both authors' career interests and in developing their professionalism.

Professor George C. Hsiao is a founder of the mathematical theory of boundary element methods. We have benefitted greatly from reading his papers. He also generously consulted with us and provided assistance on numerous occasions.

We have tremendous admiration for both individuals. To them we dedicate this monograph.

BOUNDARY ELEMENT METHODS

Goong Chen

*Professor of Mathematics and Aerospace Engineering
Texas A & M University
College Station, Texas 77843, USA*

Jianxin Zhou

*Assistant Professor of Mathematics
Texas A & M University
College Station, Texas 77843, USA*



ACADEMIC PRESS

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London San Diego
New York Boston Sydney Tokyo Toronto*

Dedication

*Dedicated to
Professor David L. Russell
on the occasion of his 50th birthday*

and

*Professor George C. Hsiao
on the occasion of his 55th birthday*

獻給我們的老師：
蕭家駒教授
及
大衛·羅素教授

Professor David L. Russell is the former Ph.D. thesis advisor of the first author (G.C.) at the University of Wisconsin-Madison. His work and personality have had a profound influence in shaping both authors' career interests and in developing their professionalism.

Professor George C. Hsiao is a founder of the mathematical theory of boundary element methods. We have benefitted greatly from reading his papers. He also generously consulted with us and provided assistance on numerous occasions.

We have tremendous admiration for both individuals. To them we dedicate this monograph.

A Thoughtful Thank You

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To: George Hsiao

From: Peter Levin and Andrew Hansen

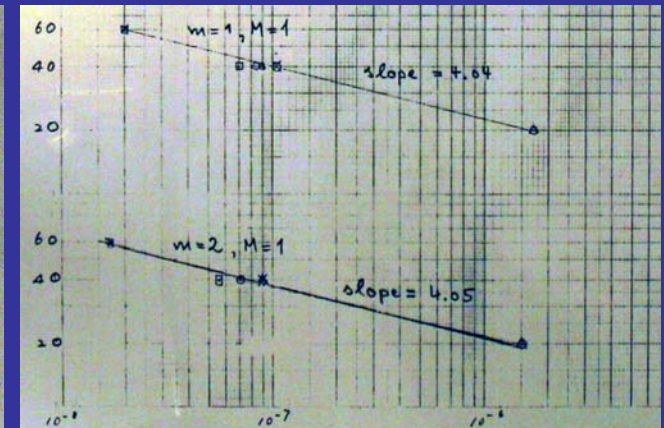
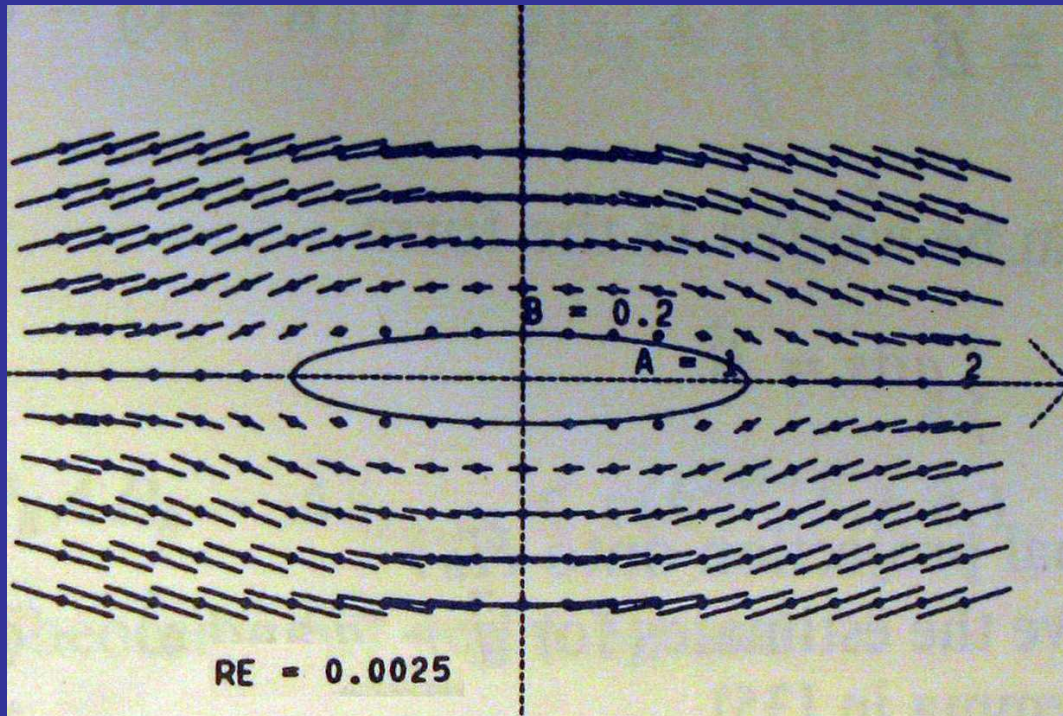


Some Numerical Experiments

UNIVERSITY OF DELAWARE

Flow Pattern

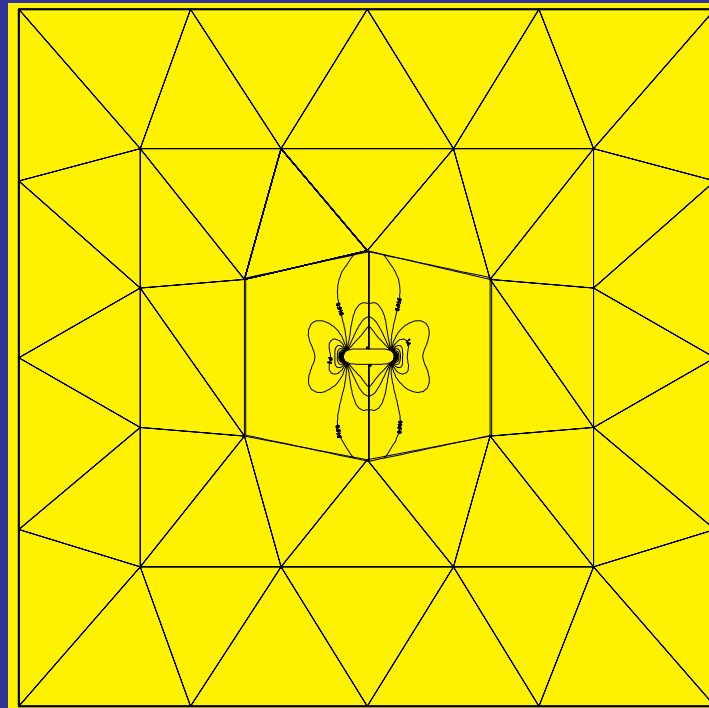
Hsiao, G. C., Kopp, P., and Wendland, W. L.,
Some applications of a Galerkin – Collocation method for
boundary integral equations of the first kind,
Math. Method. Appl. Sci. 6 (1984) 280--325.



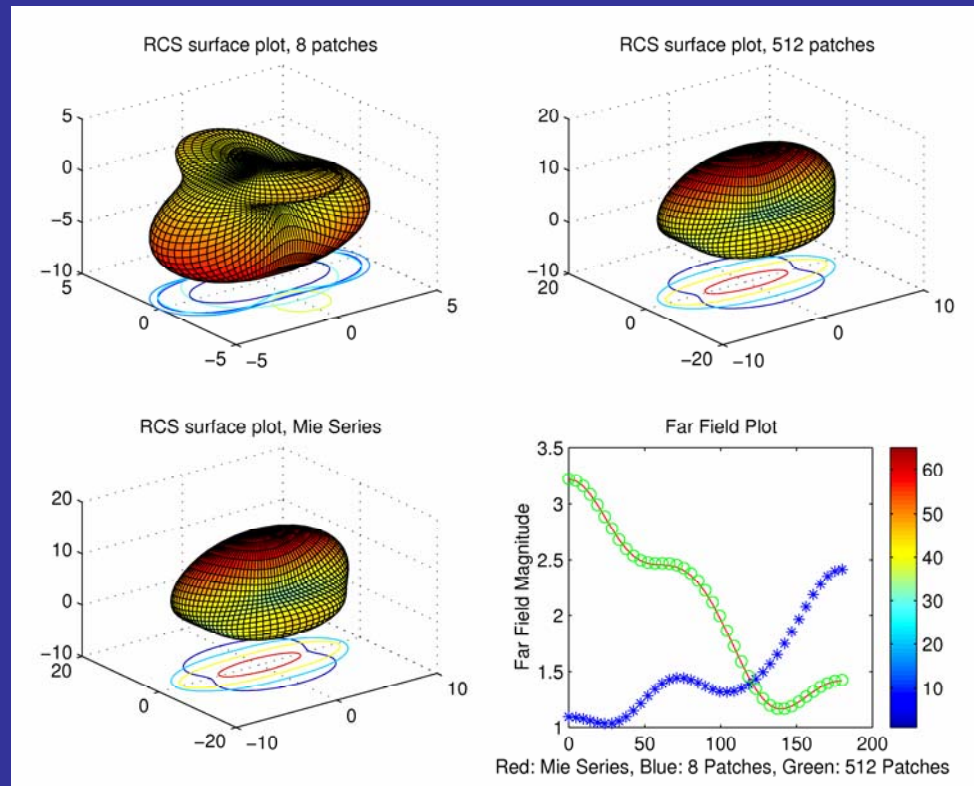
Viscous Flow Past
an Obstacle
& Absolute Error
Estimate

High Stress Gradients

G.C. Hsiao, E. Schnack, and W.L.Wendland,
Hybride coupled finite-boundary element methods for
Elliptic systems of second order,
Comput. Methods Appl. Mech. Engrg. **190** (2000) 431-485



Radar Cross Section



G.C. Hsiao, R.E. Kleinman and D.Q. Wang,
Applications of boundary integral methods in
3D electromagnetic scattering,
J. Comp. Appl. Math., **104** (1999) 89-110.

Ongoing Research: Pseudo-differential Operators (ψ dOs)

UNIVERSITY OF DELAWARE

Basic Approaches

- Recast boundary integral operators as ψ dOs.
- Analyse boundary integral equations by employing appropriate mathematical tools in ψ dOs.

The Standard ψ dO

- **Symbol Class $S^m(\Omega \times \mathbb{R}^n)$**

Def: for $m \in \mathbb{R}$, the symbol class $S^m(\Omega \times \mathbb{R}^n)$ is defined to consist of the set of functions $a \in C^\infty(\Omega \times \mathbb{R}^n)$ with the property that, for any compact set $K \subset\subset \Omega \subset \mathbb{R}^n$ and multiple-index α, β there exist positive constants $c(K, \alpha, \beta)$ such that

$$\left| \left(\frac{\partial}{\partial x} \right)^\beta \left(\frac{\partial}{\partial \xi} \right)^\alpha a(x, \xi) \right| \leq c(K, \alpha, \beta) (1 + |\xi|)^{m - |\alpha|}$$

for all $x \in K$ and $\xi \in \mathbb{R}^n$.

● **Standard ψ dO Class** $OP S^m(\Omega \times \mathbb{R}^n)$

Def: For $a \in OP S^m(\Omega \times \mathbb{R}^n)$, the set of all standard ψ dO's $A(x, D)$ of order m is denoted by $OP S^m(\Omega \times \mathbb{R}^n)$,

$$\begin{aligned} A(x, D)u &:= \mathcal{F}_{\xi \mapsto x}^{-1}(a(x, \xi) \mathcal{F}_{y \mapsto \xi} u(y)) \\ &= 2(\pi)^{-n} \int_{\mathbb{R}^n} \int_{\Omega} e^{i(x-y) \cdot \xi} a(x, \xi) u(y) dy d\xi \end{aligned}$$

for $u \in C_0^\infty(\Omega)$ and $x \in \Omega$.

Theorem 1 *The operator $A \in OPS^m(\Omega \times \mathbb{R}^n)$ is a continuous operator*

$$A : C_0^\infty(\Omega) \rightarrow C^\infty(\Omega).$$

The operator A can be extended to a continuous linear mapping from $\tilde{H}^s(K)$ into $H_{loc}^{s-m}(\Omega)$ for any compact subset $K \subset\subset \Omega$. Furthermore, in the framework of distributions, A can also be extended to a continuous linear operator

$$A : \mathcal{E}'(\Omega) \rightarrow \mathcal{D}'(\Omega).$$

George C. Hsiao and Wolfgang L. Wendland

Boundary Integral Equations

Mathematics – Monograph

August 31, 2005

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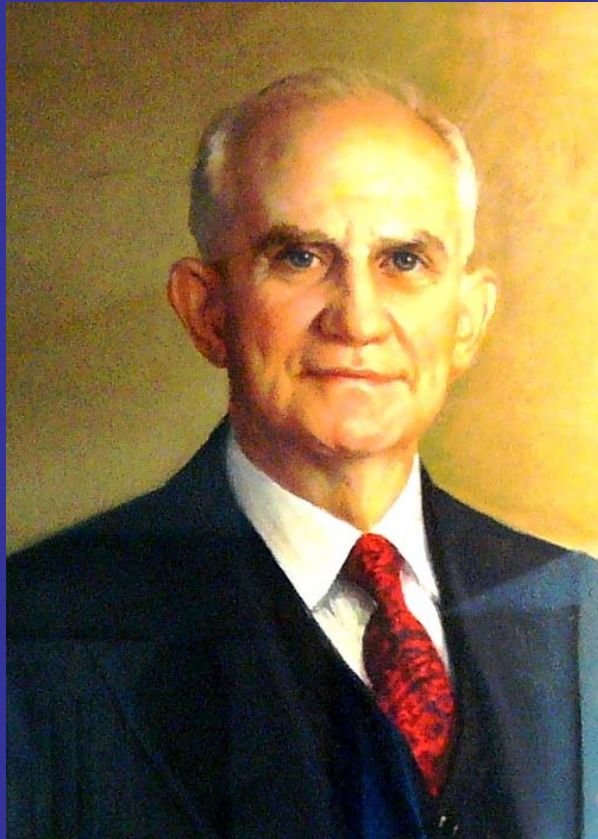
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● Concluding Remarks

UNIVERSITY OF DELAWARE

Carl J. Rees



Carl J. Rees
SERVANT OF UNIVERSITY
1920-1962
Professor of Mathematics
Graduate Dean, Provost

- Carl Rees had a distinguished career at the University of Delaware from 1920 until his retirement in 1967. He advanced through the professorial ranks to Professor of Mathematics and served as Chair of the Department of Mathematics for ten years. He was appointed Dean of the College of graduate Studies as well as Provost of the University, serving simultaneously in these important posts for a number of years.
- He served his country in both World War I and World War II and was awarded the Medal of Freedom by General H. H. (Hap) Arnold for his work as an operations analyst.

Rees Hall in 80's (Applied Math Institute)



Our Friend Ralph



Rees Hall



Mathematisches Forschungsinstitut Oberwolfach



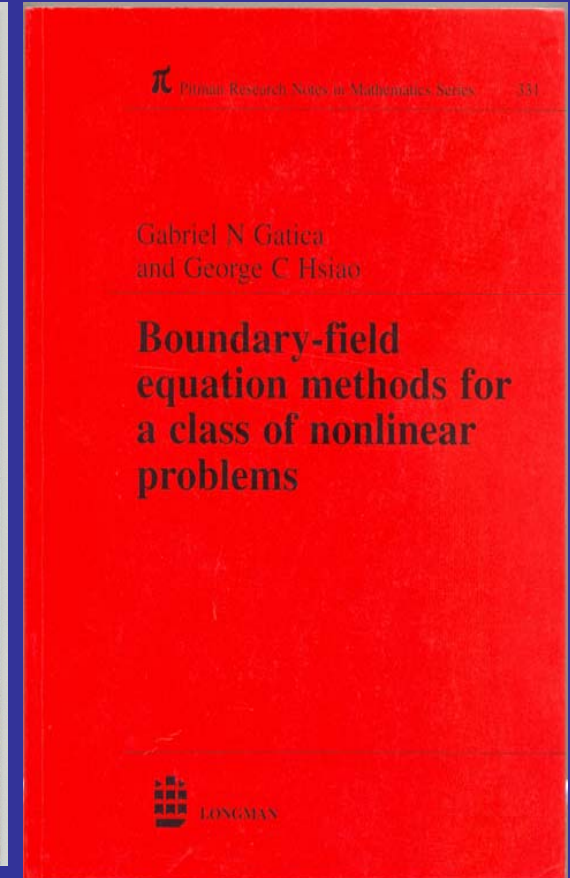
Professor Richard C. MacCamy (Carnegie-Mellon University)
Professor Wolfgang L. Wendland (Stuttgart University)

Three Generations



Prof. Manil Suri (UMBC), the author of the novel "The Death of Vishnu"
Prof. Gabriel Gatica, Chair of Numerical Math., Univ. de Concepcion

Three Generations



Prof. Manil Suri (UMBC), the author of the novel "The Death of Vishnu"
Prof. Gabriel Gatica, Chair of Numerical Math., Univ. de Concepcion

My Birthday Gift

Artificial Boundary Conditions for the Two-dimensional Wave
Equation

R.C. MacCamy*

1. Introduction

This paper represents a talk given by the author at the Conference on Boundary Elements at Oberwohlfach. It is based on joint work with T. Hagstrom and S. Hariharan, [6]. The paper is dedicated to George Hsiao in honor of his sixtieth birthday. Many years ago George was my student. Now he is the teacher.

My Collaborators and My Bosses in Math

- Thomas S. Angell
- Fioralba Cakoni
- Robert P. Gilbert
- Peter B. Monk
- Richard J. Weinacht
- Shangyou Zhang
- Ralph E. Kleinman
- Russell Remage, Jr.
- Willard E. Baxter
- L.P. Cook-Ioannidis
- Ivar Stakgold
- Philip Broadbridge



Collaborators in Engineering

Tsu-Wei Chou, Morton M. Denn, C.Y. Yang

JOURNAL OF COMPOSITE MATERIALS

Theoretical Analysis of Wave Propagation in Woven Fabric Composites

BAOXING CHEN* AND TSU-WEI CHOU**
Center for Composite Materials and
Department of Mechanical Engineering
University of Delaware
Newark, DE 19716-3144

GEORGE C. HSIAO
Department of Mathematical Sciences
University of Delaware
Newark, DE 19716-3144

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1

Creeping Flow of a Viscoelastic Liquid Through a Contraction: A Numerical Perturbation Solution

JESSE R. BLACK, MORTON M. DENN and GEORGE C. HSIAO

ABSTRACT

Creeping planar flow of a viscoelastic liquid through a contraction between upstream and downstream regions of fully developed laminar flow has been studied by combining a numerical solution with a perturbation in Weissenberg number. Computations were carried out for contraction ratios of 2:1 and 5:1 and entry half-angles of 30°, 45°, 60° and 90°. The numerical scheme requires solution of a set of singular integral equations for a density function along the flow boundary, and the stream function and all its derivatives are computed by weighted integration of the numerical density function. The region of convergence of the perturbation series is estimated to include most of the range of practical processing interest.

A recirculating corner eddy is observed for the 90° entry, but not for smaller entry angles. The velocity is more than 90% developed at the downstream entrance for all cases. The entry pressure drop and entry power requirement are weak functions of entry angle for angles of greater than 45°. The first elastic correction to the entry power requirement is obtained analytically and is shown to be small and negative.

1.1. INTRODUCTION

The behaviour of polymeric liquids has been studied extensively in viscometric shear flows, and the relationship between rheological properties and flow behaviour is well understood (e.g. Pipkin and Tanner, 1972). Many of the flows of practical interest in polymeric processing are not viscometric flows, however, nor can they be considered to be small perturbations about viscometric flows. The manner in which fluid properties influence the kinematics and stress field is poorly understood in a number of such

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SOME MATHEMATICAL CONCEPTS RELATED TO STOCHASTIC SPECTRUM ANALYSIS

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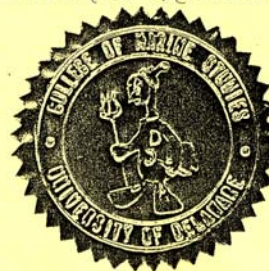
is hereby recognized and highly commended for assistance in achieving sustained excellence in research, education, and public service leading to the designation of the University of Delaware as a Sea Grant College. For special services to the University, the State of Delaware, and the Nation therefore this certificate is granted this third day of May, nineteen hundred seventy-seven at Newark, Delaware.

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