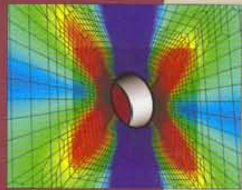


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Chapter 12

Boundary Element Methods: Foundation and Error Analysis

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1 INTRODUCTION

In essence, the boundary element method (BEM) may be considered as an application of finite element method (FEM), designed originally for the numerical solutions of partial differential equations (PDE) in the domains, to the boundary integral equations (BIE) on closed boundary manifolds. The terminology of BEM originated from the practice of discretizing the boundary manifold of the solution domain for the BIE into boundary elements, resembling the term of finite elements in FEM. As in FEM, in the literature, the use of the terminology *boundary element* is in two different contexts: the boundary manifolds are decomposed into boundary elements, which

are geometric objects, while the boundary elements for approximating solutions of BIEs are actually the finite element functions defined on the boundaries. Looking through the literature, it is difficult to trace back one fundamental research paper and the individuals who were responsible for the historical development of the BEM. However, from the computational point of view, the work by Hess and Smith deserves mention as one of the cornerstones of BEM. In their 1966 paper (Hess and Smith, 1966), boundary elements (or rather surface elements) have been used to approximate various types of bodies and to calculate the potential flow about arbitrary bodies. On the other hand, the paper by Nedelec and Planchard (1973) may be considered as a genuine boundary element paper with respect to the variational formulation of BIEs. Other early contributions to the boundary element development in the 1960s and 1970s from the mathematical point of view include Fichera (1961), Wendland (1965, 1968), MacCamy (1966), Mikhlin (1970), Hsiao and MacCamy (1973), Stephan and Wendland (1976), Jaswon and Symm (1977), LeRoux (1977), Nedelec (1977), and Hsiao and Wendland (1977), to name a few.

The BEM has received much attention and gained wide acceptance in recent years. From 1989 to 1995, the German Research Foundation DFG installed a Priority Research Program 'Boundary Element Methods', and the final report appeared as a book (see Wendland, 1997). There has been an increasing effort in the development of efficient finite element solutions of BIEs arising from elliptic boundary value problems (BVP). In fact, nowadays, the term BEM denotes any 'efficient method' for the approximate numerical solution of these boundary integral equations.

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