Lev Arkad'evich Kalužnin (1914–1990)

A Short Biography

Lev Arkad'evich Kalužnin was born in Moscow, Russia, on 31 January 1914. Shortly after his birth, his parents divorced and his father, Arkadiĭ Rubin, a well-known businessman, moved to England. The child was brought up by his mother, Maria Pavlovna Kalužnina. She came from an old noble family many of whose members had become prominent figures in Russian culture, education, and the Arts. She passed her cultural values on to her son, especially her deep love for literature and music. She had a great influence on Lev Arkad'evich throughout her life – sometimes markedly so.

During the revolution of 1917 and the civil war which followed, mother and son lived in Petrograd (now Saint Petersburg). In 1923, they moved to Germany. To make ends meet, Maria Pavlovna worked as a governess. In 1925, Lev entered a secondary school (Realschule) of high academic standing, from which he would graduate in 1933. The school provided a solid background in mathematics, including topics in the foundations of analysis, differential equations and complex variables. In the fall of 1933, he started as a freshman at the Humboldt University of Berlin, where he would spend the next three years. While at Humboldt, he was greatly influenced by I. Schur, whose lectures on algebra shaped the youth's mathematical interest. In 1936, Lev moved to Hamburg, where, at the University of Hamburg, he attended lectures of E. Artin and E. Hecke and seminars of H. Zassenhaus and other famous mathematicians. It was here where he obtained his first research result – a generalization of what is today a well-known theorem of A. G. Kurosh on the classification of Abelian groups. In the spring of 1938, Lev Arkad'evich moved with his mother to France, where he started attending lectures at the Sorbonne about a year later.

The war and occupation of Paris by German troops forced Kalužnin to terminate his mathematical studies. To make a living during these difficult times, he attended a vocational school and became an electrician. On 22 June 1941, his life changed dramatically – as did the lives of many Soviet citizens, who, like Kalužnin, were interned and sent to a camp in Compiègne near Paris. In the beginning, conditions in the camp were tolerable and some prisoners, being specialists in certain fields, would entertain themselves by lecturing to others on diverse subjects. (In fact, in later years, Lev Arkad'evich could still recall great lectures he had heard there on world history, Roman law, etc.). During this time he did some research in Galois theory. In March 1942, he was transferred, as one of a group of prisoners, to a concentration camp in Wahlsburg. There, the horrors of camp life could be felt to their fullest extent. If not for the devotion

and efforts of his mother, who found ways of surreptitiously sending him food during this period, Kalužnin may not have survived to see the liberation of the camp by American soldiers.

In the spring of 1945, Kalužnin returned to Paris. For a while he worked as a translator for the Soviet Embassy, and soon after he returned to his studies in mathematics. He worked for the CNRS, published a series of papers on the structure of Sylow p-subgroups of symmetric groups, and in 1948 he defended his doctoral thesis on the same topic. The following three years were extremely fruitful: he published several fundamental papers, collaborated with M. Krasner, and presented his results at seminars and conferences.

At about this time, Lev Arkad'evich and his mother had decided to return to the USSR. In response to their application, Soviet authorities asked him to spend some time in East Germany, where there was an acute shortage of scientists. This is why in 1951 Kalužnin began working at the Humboldt University in Berlin – first as a lecturer, and later, after habilitation (*Stable automorphism groups*), as a full professor. At the same time he held a research position at the Mathematical Institute of the (East) German Academy of Sciences.

In 1955, Lev Arkad'evich returned to the U.S.S.R. Through the recruitment efforts of mathematicians B. V. Gnedenko and G. E. Shilov he was given a professorship at Kiev State University, an appointment he would hold for 31 years. At this time, the University, though steeped in tradition as one of the finer mathematical research centers in the U.S.S.R, was in a relatively poor state. Memories of Stalin's recent atrocities were fresh in people's minds, and there was a pervasive atmosphere of political denunciation. Many good faculty members were forced to leave. In 1957 Kalužnin defended his postdoctoral thesis (a Soviet version of habilitation) on the topic '*Sylow p-subgroups of symmetric groups. Complete products of groups. Generalizations of Galois theory*'. With his defence he became, in all probability, the only person to have ever received the highest degree in mathematics from three different countries – even though he had never completed his formal education! (In France, permission had been granted by special committee.)

In 1959, Kalužnin became the Head of the Department of Algebra and Mathematical Logic, a department created as a result of Kalužnin's own initiative. He became interested in mathematical linguistics and played an important role in the creation of the Department of Mathematical Linguistics at Kiev State University. A partial list of his activities and accomplishments during the decade 1960–1970 includes conducting research, teaching at Kiev State University and the Kiev Pedagogical Institute, acting as a consultant for the Department of Mathematical Linguistics, serving as a senior researcher at the Institute of Cybernetics of the Ukrainian Academy of Sciences, organizing series of public lectures on mathematics, and serving as a member on editorial boards of several scientific journals. In 1962, he married Zoya Mikhailovna Volotskaya, a well-known

linguist. They had two children, but lived apart for most of the time there-after.

In 1968, several friends and students of Kalužnin signed a letter condemning the closed political trials that were then commonplace in Ukraine. In 1970, the political climate deteriorated still further. Having always been perceived as a sort of 'alien', Kalužnin was forced to abandon his position as Head of the Department of Algebra and Mathematical Logic, though he retained his professorship at Kiev State University until 1985. He was also denied permission to attend numerous conferences abroad to which he had been invited, and his only contact with Western mathematicians was now limited to correspondence through mail. During these years, he devoted most of his time to his students, to his research activities on permutation groups, and to his newly cultivated interest in computer algebra. In 1984, due to deteriorating health, Lev Arkad'evich relinquished his teaching duties, and in 1985 his position within the faculty was changed to that of a 'senior researcher'. At the same time, his son Mikhail returned to religious involvement, quit Komsomol, and became the target of constant attacks. (There was no chance that young Mikhail's deeds would go unnoticed by the communist authorities as he was at this moment graduating from the same Faculty of Kiev State University, so his actions came under close scrutiny.) All this contributed heavily to Kalužnin's retirement and to his subsequent move to Moscow. As time passed, Kalužnin's state of health became worse, and one day he became severely injured as the result of an accident. Lev Arkad'evich died on 6 December 1990. He is survived by son Mikhail and daughter Olga. (For additional biographical details see [S1, S2].)

Main Professional Activities

We will divide L. A. Kalužnin's professional output into three parts: (I) Research into algebra and algebraic combinatorics; (II) Mathematical linguistics, automata theory, applications of computers in algebra; and (III) Teaching and writing.

I. RESEARCH IN ALGEBRA AND ALGEBRAIC COMBINATORICS

As a researcher, Kalužnin is best known for his work in group theory and permutation groups. Papers [4–8, 11, 12, 25] are devoted to studies of Sylow p-subgroups of symmetric groups and their generalizations. In the case of symmetric groups of degree p^n, these subgroups – call them P_n – were constructed from cyclic groups of order p by taking their wreath product (or, in Kalužnin's terminology, 'complete product'). This allowed one to represent the elements of P_n as certain sequences of polynomials, called tables, and to replace computations in groups by computations in certain polynomial algebras over the field of p elements. Despite the fact that the earliest, simple applications of wreath products of permutation groups can be traced to C. Jordan, W. Specht and G. Pólya, it

was Kalužnin who first developed special computational tools for this purpose. Using his techniques, he was able to describe the characteristic subgroups of P_n, their derived series, their upper and lower central series, and more. These results have been included in many textbooks on group theory.

In [8, 12, 25], table representations were used to study the Sylow p-subgroup P_∞ of the full isometry group of 'complete Cantor space' (isomorphic to the space of all p-adic integers with usual metric). Among his many results, Kalužnin established an important property of P_∞, namely that it is a universal object, with respect to embeddings, in the class of all groups which have a series of trivially intersecting subgroups all of whose factors have order p. Nowadays studies of this group and its subgroups form an important topic in the theory of automata transformations – a new branch of modern algebra which has many interesting and far-reaching applications to group theory, combinatorics, geometry, mathematical logic, and dynamical systems.

Kalužnin was also the first to introduce the wreath product of abstract groups [16] and the wreath product of an infinite family of groups [12]. His constructions could be applied to group extensions. In this regard, we mention the well-known theorem of Krasner and Kalužnin [19, 23, 24] concerning the embedding of a group with a subnormal series into the wreath product of the factors of the series. This theorem is widely used in the theory of group varieties, combinatorial group theory, and permutation group theory.

The wreath product of groups can act as a permutation group in various ways [90]. One such action is the exponentiation of two groups, introduced by F. Harary, where the wreath product acts on the set of all functions from one set to another. In [76, 90], a condition for the exponentiation of two groups to be primitive was found. Several applications of wreath products, to mathematical logic and mathematical chemistry are discussed in [88, 109]. The monograph [113] is devoted to work on wreath products conducted by Lev Arkad'evich, his students, and his followers.

Among Kalužnin's other significant contributions to group theory, we mention his work on stable automorphism groups [21, 22, 26], the structure of the variety of n-abelian groups [54], a classification of metabelian groups [27], work on locally normal groups of higher categories [49], and characterizations of the maximal subgroups of the symmetric and alternating groups [69, 88, 97].

Another area of algebra which had always attracted Kalužnin's interest is abstract Galois theory (see the monograph [89] and references therein). His ideas deeply influenced further investigations of relational and functional systems via a Galois correspondence between operations and relations. His first papers in this area were devoted to Galois theory of normal extensions of fields [3, 10, 13, 32]. The ideas of this work were later employed by N. Jacobson in his Galois theory of arbitrary finite extensions of fields. Developing further the methods of abstract Galois theory which had been initiated by Krasner, Kalužnin and his

students were able to establish a Galois correspondence between Post algebras and Krasner algebras [62, 63]. Further studies related to permutation groups led to the construction of the first infinite two-parameter series of maximal uniprimitive subgroups of symmetric groups [69]. This sparked enormous activity in the (then) newly developed research field which has since come to be called algebraic combinatorics. (Our use of the term 'algebraic combinatorics' is here restricted to the study of cellular rings, i.e. coherent configurations, and the study of groups via their permutational actions.)

In 1933, I. Schur [S3] introduced the notion of a V-ring (from the German 'Vertauschungsring', meaning centralizer ring) as well as an important subclass of such rings, called S-rings. (Kalužnin, himself, was responsible for the term V-ring; the term S-ring had been introduced by H. Wielandt.) Schur successfully applied S-rings to the study of certain kinds of permutation groups. Kalužnin fully understood the potential of the concepts of V-ring and the Galois correspondence between the subgroup lattice of a group and the cellular subring lattice of a V-ring associated with the group. Moreover, he foresaw the importance of these concepts as applied to the study of various combinatorial objects with rich symmetries. Though most of the research in this direction has been done – and indeed, is still *being* done – by Kalužnin's students and others (see [S4] for an up-to-date status report), it was Kalužnin's results, his extraordinary vision, and his encouragement that laid the foundation for these future successes.

We also mention that Kalužnin worked in the area of geometrical algebra (see [50, 51]), particularly on arrangements of subspaces in Euclidean and unitary spaces.

II. MATHEMATICAL LINGUISTICS, AUTOMATA THEORY, APPLICATIONS OF COMPUTERS IN ALGEBRA

Though Lev Arkad'evich had never considered himself to be an expert in any of these fields, his intellectual spectrum was broad and he did not hesitate to do research in areas outside his main fields. He believed in the fruitfulness of cooperation among people representing different mathematical disciplines, and that there were certain advantages to be had by 'amateurs' having solid experience in another field. Often the areas he chose to concentrate on were little known to the scientific communities of the former Soviet Union.

Lev Arkad'evich was successful in finding talented collaborators and in organizing scientific groups for such enterprises. Through his public lectures and expository articles, he was able to introduce large audiences of mathematicians to areas in which he had gained an interest. Many of Kalužnin's ideas and results in these areas served as a basis for continuing investigations. Publications [41, 42, 44, 45, 64] are related to mathematical linguistics, [48, 55] – to automata theory, and [85, 102, 105] – to applications of computers in algebra.

III. TEACHING AND WRITING

L. A. Kalužnin was an outstanding teacher. His lectures were inspiring. He was sometimes a bit impatient with the details of proofs, concentrating instead on presenting the broad picture of the mathematical terrain with regard to events that had taken place. At this he was hugely successful. He liked to discuss questions in their proper historical context, to motivate new concepts, and to stress the meaning and importance of a particular result.

Lev Arkad'evich had a gift for recognizing sparks of talent and potential in his students at a very early stage, often during their first semester at Kiev State University. He would then attempt to monitor their progress and keep them under a 'watchful eye'. He was extremely patient in building the skills of these students, always stressing the importance of breadth in mathematical education. Kalužnin was very adept at attracting students to areas in which he himself had an interest. He cared about mathematical education at all levels – from elementary school to graduate programs – and he did a lot to improve its quality through seminars, lectures, workshops and writing.

Some of Kalužnin's books are masterpieces of mathematical exposition, having been translated into several languages and published in several countries [46, 47, 61, 73, 77, 81, 84, 93, 94, 101, 104, 108, 111]. His works [68] and [73] serve as excellent textbooks. His German-Russian mathematical dictionary [38, 60, 98] is one of the finest ever written, and is still widely used today. All told, Kalužnin is the author and/or coauthor of more than 110 publications, including two monographs and 10 textbooks and handbooks.

Lev Arkad'evich was a cofounder of the Boarding High School at Kiev State University, a school with a specialization in mathematics and physics. In the late 50's, he participated in organizing mathematical contests for high school students. At university level, he served as an advisor to no fewer than 21 PhD students, many of whom have gone on to become very well known and highly respected in their fields.

General

Despite the absurdities and humiliations of Soviet life, with its constant dependencies on the whims of the ignorant and/or indecent who were in power, Lev Arkad'evich was never heard to regret his return to the USSR. Trying to avoid direct political involvement, he nevertheless found ways to make his position on social and political issues known, or to respond to pseudo-scientific political propaganda (see, e.g., [29]).

Lev Arkad'evich fully realized the extent of his accomplishments as a researcher, teacher, writer, and founder of scientific groups, and he well understood his impact on mathematical education. It is hard to overestimate the influence he has had on the lives of his many students. To those who grew up in the

U.S.S.R., his genuine respect for the individual – an outgrowth of his Westernized way of thinking – was a new and unusual phenomenon. Looking back, we now realize that besides being a teacher and advisor, he was, for many of us, a link between our Soviet reality and the unknown and mysterious Western World. How many people in Kiev spoke fluently in several languages, lived in great European cities, and knew personally the most outstanding mathematicians of the twentieth century? And if so, how many of them would have been willing to share there experiences, as did Kalužnin, with 18-year old freshmen on walks home after seminars? For some of us, these seminars survived for the full five years that comprised our undergraduate education.

Some of the material discussed at these seminars was quite elementary, and so much part of the standard curriculum in algebra at all Soviet universities. Nonetheless, much of it was not introduced until the second or third year of study. Kalužnin would assign each first-year student the task of presenting portions of this material to a general seminar audience. In doing so, he not only exposed these students to new mathematical concepts; he was able to observe them in action, to see how they reacted under the pressures of presenting 'unfamiliar material' in a seminaral format. Never wishing to destroy a student's confidence, Kalužnin remained vigil throughout these lectures, always ready to throw a 'lifejacket' to a floundering student.

Only now, after having become teachers ourselves, can we fully appreciate Lev Arkad'evich's patience and dedication. How many of us could sit, listen attentively, and provide encouragement during a freshman's presentation on the basic properties of congruences of integers? What if the presentation were to take three seminar meetings?

A few remarks on personality. Lev Arkad'evich used to be a heavy smoker, burning up to three packs of cigarettes a day, even during his lectures. He quit smoking on 1 January 1970, and never smoked again. He was well versed in classical music, classic philosophy and Western prose, all of which he loved passionately. He did not care much for poetry, saying that 'a bear, probably, stepped on my poetic ear'. Kalužnin was associated with many members of Kiev's intellectual elite and he often told his students many interesting things that he had learned from them. He had a definite tendency to get 'carried away' with new ideas – for example, organizing mathematical gatherings in a cafe (similar to a famous cafe in L'vov during the 30's), starting 'musical evenings' in the students' dorm, or discussing the contents of a new book he would like to write. If the weather were especially fine, he would offer the suggestion to 'interrupt the educational process' and invite the entire class outside. (Remembering the time and place, this was certainly an unheard of practice.)

Lev Arkad'evich liked to dress well and usually adorned himself in the Western clothing he had brought back with him on his move to Kiev, or which had been sent to him by friends in Western Europe. At the conclusion of a lecture, his clothes would be entirely covered with chalk; at least this was the case until the

early 70's when, after visiting the G.D.R., he had returned with a white smock, the purpose of which was to protect his clothing during lectures. This unconventional attire prompted the joke among his students that Kalužnin is the 'only real doctor of the Department'. It also enhanced the perception of Lev Arkad'evich as being the solitary 'white crow' in an otherwise uniform flock . . . .

On the other hand, Kalužnin sometimes found it difficult to part with old clothing. One day, a well-meaning secretary decided to help him by asking someone to dispose of a dilapidated jacket he liked to wear. The jacket, which normally hung on the wall of his office, was unceremoniously dumped into a trash can in the men's room. To the secretary's dismay, she later saw a smiling Lev Arkad'evich, jacket in hand, puzzled by the mystery of how it could have found its way into the men's room trash can. The very next day she disposed of it herself, only *this* time to a trash can in the ladies' room.

Another funny incident. Once, by accident, Kalužnin left the department office wearing the coat of his student, oblivious to the fact that it was at least 25 centimeters longer than his own coat! Half an hour later he returned with a very grim face. Alas, Lev Arkad'evich had discovered a hole in the pocket of his(!) coat, and through this hole, he now deduced, he must have lost his wallet.

Lev Arkad'evich liked and valued good red wines and also good beer (the latter being an extreme rarity in the former Soviet Union). He was deeply respected and loved by most who knew him. For us, as well as for many of our friends, his strong personal influence has had a profound effect on our lives. His clever and gentle humour, his aristocratically cultivated manner and free spirit, his extraordinary friendliness and openness to his companions, his clear and strict rejection of all forms of discrimination (be they nationally, religiously or politically based), his everyday deeds and reflections, his pure heart and his intellectual soul – all of these have contributed to form an image of Lev Arkad'evich Kalužnin which lives eternal in the minds of his disciples. We wholeheartedly believe that the spark of his great spirit will not disappear with our generation.

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F. G. LAZEBNIK,   R. PÖSCHEL,
V. A. USTIMENKO,   V. A. VYSHENSKIĬ

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