The Problem of Transaction Costs

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Abstract: Mechanism design provides a general paradigm for deriving legal rules and institutions that implement social objectives as equilibrium outcomes of interactions among strategic individuals with private information and endogenous transaction costs. This paper designs mechanisms that generalize Ronald Coase’s farmer-rancher example of nuisance law from his seminal article, “The Problem of Social Costs,” and Judge Learned Hand’s negligence liability tests from his classic opinions in T. J. HOOPER (1932) and CARROLL TOWING (1947). Mechanism design encompasses the Pigouvian-social-planner and Coasean-transaction-cost paradigms as special cases, while accommodating alternative societal objectives, various equilibrium concepts, endogenous transaction costs, strategic agents, private information, truthful revelation of private costs and benefits, incentive compatibility, voluntary participation, and budgetary feasibility. These features extend previous paradigms and test their robustness in more sophisticated economic environments. The mechanism design paradigm provides modern theoretical foundations for economic analysis of law.

Keywords: Mechanism design, Coasean transaction costs, Hand test, bilateral externalities, social costs, nuisance, negligence.

JEL Codes: K0, D82, D23, K11, K13, C7

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1.0 Introduction

This paper designs two new liability mechanisms that implement efficient legal outcomes in nuisance and negligence law, to illustrate a general mechanism design approach to the economic analysis of law. These mechanisms incentivize strategic parties to truthfully report their private information about the benefits and costs of their activities to a court or other authority. The court uses liability rules to align the parties’ mutual best responses or dominant strategies with socially optimal outcomes in equilibrium. The incompatible activities mechanism for nuisance law causes the parties to truthfully report their private net benefits and costs of alternative activity levels. It designs liability schedules so that the parties choose their joint surplus maximizing activity levels as their dominant strategies. The (benighted Hand) negligence mechanism, enables an uninformed authority to apply liability rules that cause the parties to report their private costs of care and choose bilateral care levels that minimize their joint expenditures on precaution and expected accident costs when their care levels have different effectiveness in preventing joint losses.

While these mechanisms are novel and significant in their own right, the main point of this paper is to elaborate a general mechanism design paradigm for economic analysis of law. The mechanism design framework treats transaction costs as endogenous costs of credibly conveying the parties’ private information about preferences, costs and benefits, and other characteristics, when they behave strategically subject to incentive constraints and budgetary feasibility. The framework specifies the initial distribution and subsequent exchange of information that is necessary to determine efficient outcomes. Legal rules and institutions arise endogenously to govern legal interactions and coordinate decentralized private decision-making so that the parties achieve socially desirable outcomes in game-theoretic equilibrium. Endogenously designed rules and institutions cause the equilibrium outcomes to coincide with precisely stated social objectives, such as joint surplus maximization or cost minimization.¹

Ronald Coase’s seminal paper, “The Problem of Social Cost” (1960), developed the Coasean transaction cost paradigm that underlies much of the existing economic analysis of law, especially in property, torts, and contracts. The Coasean approach allocates property rights to mitigate the effects of transaction costs on private negotiations and facilitate the transfer of rights to the party who values them most. It incorporates Judge Learned Hand’s (1932, 1947) mathematical formulation of negligence and product liability law that sets the standard of care for activities that

¹ Mechanisms can achieve other social goals, such as fairness or Rawlsian justice.
impose external risks of harm at the social cost-minimizing level. The approach also applies to allocation of contractual rights and obligations when contractual drafting costs prevent the specification of all possible contingencies (see, e.g., Steven Shavell 1984).

While the Coasean transaction cost approach has produced a rich literature on law and economics, mechanism design provides a modern economic framework with much greater theoretical coherence. It reproduces previous Coasean results as special cases while systematically extending the analysis to cover new informational issues, heterogeneous agents, strategic behavior, and incentive compatibility. It handles a wide variety of social objectives while requiring that the social objectives be attained as outcomes of formal equilibrium concepts. The Coasean literature often includes some of these features, but generally does not take full advantage of the theoretical advances from mechanism design.

In particular, I argue that the theory of law and economics should systematically adopt the following characteristics. Transaction costs should be modeled endogenously as arising from the need to exchange credible information to determine optimal outcomes and coordinate decentralized decisions. Likewise, the legal rules and institutions should arise endogenously as the formal solution to the problem of aligning equilibrium outcomes with social goals. The initial distribution and subsequent exchange of information should be explicitly modeled to determine whether the endogenous rules elicit truthful revelation of private information. The social goals and the equilibrium concepts should be specified formally with necessary and sufficient conditions for the equilibrium outcomes to produce the social objectives.

I also argue that there is a strand of unproven or erroneous results in the Coasean law and economics literature that stem from the informal nature of the Coase “theorem.” Economic analysis of law should not credit the unproven assertion that unspecified private bargaining necessarily overcomes ad hoc transaction costs and achieves efficient results under exogenously determined legal doctrines. Common law doctrines often implicitly assume that judges are perfectly and costlessly informed, and then fashion “efficient” rules under these assumptions. Reformulating such models as problems of mechanism design tests the robustness of previous results in well-specified economic environments.²

²For example, the industrial organization literature on economics of contracts uses mechanism design and the economics of information to challenge the law and economics notion that contract doctrines are efficient. It argues that legal doctrines undermine ex-ante efficient contracts by disallowing contractual bans on renegotiation (see, e.g., Jean Tirole, 1999, and Alan Schwartz and Robert E. Scott, 2003). Likewise, the mechanism of John Moore and Rafael Repullo (1988) shows that incomplete contracts can achieve efficiency without requiring default provisions, which are often counter productive. In this paper, I argue that mechanism
Mechanism design has produced spectacularly successful rules and institutions in real-world cases such as the National Resident Matching Program for hospital residencies and the Federal Communications Commission (FCC) auctions of radio spectra. The Resident Matching program efficiently matches about twenty-five thousand applicants with medical training residencies every year based on mutual preference revelation. The FCC has conducted dozens of auctions raising billions of dollars in revenues under conditions that require individual- and bundle-specific pricing of alternative bundles of the spectrum (see Alvin Roth, 2003). Design of legal mechanisms for resolution of disputes using incentives for truthful revelation of private and subjective information holds similar promise. Estimates of litigation costs by the insurance industry, not including settlements and judgments, exceed $288 million annually (Joni Hersch and Kip Viscusi, 2007). Economic analysis of law should seek rules that reduce litigation costs by incentivizing truthful reporting of private information.

To illustrate the mechanism design paradigm, this paper develops mechanisms that replicate the results from Coase’s farmer-rancher example of nuisance law and Hand’s standard of care for negligence and products liability law when there is perfect information. Then it derives new nuisance and negligence rules in the richer economic environment of mechanism design that implement social objectives as equilibrium outcomes when agents have private information about their own costs and benefits associated with activities that produce multilateral externalities.

Mechanism design is more than an extension of Coasean transaction-cost analysis because it reveals strict limitations on previously accepted results and it provides a broader framework for endogenously modeling legal institutions that regulate strategic behavior with private information.

2.0 The mechanism design approach to law and economics

The fundamental problem in law and economics is how to encourage an economically rational man to behave as a legally reasonable man. Rational people systematically pursue their self-determined objectives, subject to the economic and legal constraints that their environments impose on them. They frequently behave opportunistically, meaning that they take advantage of opportunities to further their own objectives even when their actions potentially impose large losses on others. Legally reasonable people, on the other hand, act with due regard for others, and weigh the personal benefits of their actions against the costs they impose on others. While they pursue their own objectives, they refrain from opportunistic behavior when their gains are small.
relative to other people's losses. Law is the social contract by which rational people give up the freedom to do whatever they want in exchange for a social agreement to behave reasonably. Whenever people behave reasonably rather than opportunistically, the net welfare of society improves.

There are many ways in which the law encourages rational people to behave reasonably. Property law forbids unreasonable interferences with other people's enjoyment of their property, and uses the power of the state to defend these rights. Tort law establishes reasonable standards of care that people have a duty to observe when acting in ways that may potentially harm others. Contract doctrines promote mutually beneficial exchanges by requiring rational people to fulfill their commitments so that others can rely on their performance. Criminal law punishes people for egregious violations of the physical integrity or property rights of others in a way that that threatens the peace and security of society.

The central concern of mechanism design is how to create a system of incentives, communication, and behavior so that individuals who pursue their private goals and make decentralized choices can achieve socially desirable outcomes in equilibrium from which no one wishes to deviate. The social goals have various mathematical specifications, such as maximizing the joint surplus of property owners with incompatible land uses, minimizing the social cost of accidents, maximizing joint profits from contracts with reliance expenditures (contract specific investments), or minimizing the enforcement, punishment, and victimization costs associated with crime. Social objectives can also include mathematical formulations of fairness or justice. Rationality is the systematic pursuit of private goals, usually utility or profit maximization, subject to budgetary, technological, and legal constraints. Mechanism design specifies an exchange of information and strategic actions together with a set of rules that impose liability, compensation, and/or penalties to modify the agents' payoffs so that their rational strategies align in equilibrium with reasonable behavior.

The economic environment for mechanism design has the following elements:

- A social objective is some well-defined concept of the social good, e.g., profit maximization, consumer or producer surplus maximization, fairness, or Rawlsian justice. The social objective is exogenous to the mechanism.
- The agents (or parties) are a group of decision makers who pursue their own individual

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3 The Rawlsian concept of “justice” is to maximize the well being of the least well off member of society. One formulation of “fairness” is that no agent prefers another’s consumption level that is affordable at a level of work effort that the agent is willing and able to provide, given his level of skill.
objectives while interacting with others.

- The agent types specify their individual characteristics, which may or may not be observable by others. The types usually include the agent preferences, and may also include income, costs, benefits, skill, and other heterogeneous characteristics. A type profile is an n-tuple with a realized type for each agent that is drawn from his type space.

- The state of nature refers to exogenously determined conditions of the environment that may occur randomly according to a probability distribution.

- An information structure specifies what agents know about each other’s types and the states of nature. Information may change as agents interact, learn, and events unfold.

- An equilibrium concept is an explicit statement of the conditions that the agents’ strategies must satisfy to be considered a stable outcome, in the sense that agents cannot improve their individual objectives by deviating from their equilibrium strategies. The most common equilibrium concepts are dominant-strategy, Nash, sub-game perfect, and Bayesian equilibria.

- Implementation of the social objective using an equilibrium concept requires that the set of socially optimal outcomes is the same as the set of equilibrium outcomes, for all possible combinations (realizations) of agent types. This is a demanding requirement that enforces a mathematically rigorous relationship between decentralized private behavior and the attainment of social objectives.

- The mechanism consists of
  
  a) an exchange of information and strategic actions (message spaces), and
  
  b) a set of rules (outcome functions) that map the information and strategies into individual outcomes.

The mechanism is endogenously designed to assure that it implements the social objective as the result of a specific equilibrium concept.

- An authority is an entity that oversees the mechanism and has the ability to enforce the outcomes that the mechanism associates with the agents’ choices of messages and strategic actions. An authority is distinguishable from a Pigouvian social planner because it does not typically have information or expertise except as provided by the agents.

[4] The authority may be (inter alia) a self-enforcing institution (e.g., a competitive market) or agreement (e.g., an international treaty), a set of social norms, an employer, a private institution (e.g., a corporation), a legislature, an administrative agency that enforces rules, or a court backed by the coercive power of the state.
3. The problems with Coasean transaction-cost analysis

Coase’s seminal article, “The Problem of Social Cost,” emphasizes five major points. First, externalities result from mutually incompatible activities rather than one party’s wrongful activities that harm innocent victims. Second, in the absence of transaction costs, private negotiations between parties engaged in incompatible activities will achieve efficient outcomes provided the initial rights to activity levels are clearly delineated. Third, if transaction costs inhibit private negotiation, the law should delineate the parties’ rights in a way that facilitates efficient outcomes by mitigating the effects of transaction costs. Fourth, courts often ignore or do not know the private benefits or costs of activities associated with externalities or public goods, so they may be unable to determine the optimal activity levels. Fifth, decentralized solutions in which the parties to a dispute negotiate outcomes subject to clearly delineated rights and duties are generally preferable to those imposed by a third party, such as a court or social planner.

The Coasean idea of assigning legal rights and obligations to facilitate socially desirable outcomes through decentralized decision making is fundamental to economic analysis of law. For the following reasons, however, the Coasean paradigm is an inadequate theoretical framework for law and economics.

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5 This was a controversial argument initially, but is unobjectionable from the mechanism design point of view, provided that the model specifies who determines the activity levels and who experiences the costs and benefits that result.

6 Initial responses to the Coase theorem (from 1960-1980) largely focused on the validity of this claim, particularly Coase’s original formulation that the bargaining outcome was invariant to the initial distribution of rights. This “invariance version of the theorem” is false and is now repudiated by most Coaseans. His article clearly acknowledges that the no-transaction-cost version of the “theorem” is unrealistic, and is only intended to emphasize that transaction costs, which inhibit private bargaining, should be the primary focus of legal concerns with externalities (see, Robert Cooter, 1982).

7 This version is known as the “normative Coase theorem,” and is Coase’s essential and enduring contribution (Cooter 1982). Its prescriptive power, however, is greatly diminished when economic analysis of law does not model bargaining outcomes as equilibrium results of interactions among strategic agents with endogenous transaction costs.

8 Mechanism design invalidates this claim as a matter of economic theory. It designs incentive systems that elicit truthful revelation of private information. The mechanisms of Duggan and Roberts (2002) and Walker (1978), for example, specifically solve the theoretical problem of determining optimal levels of externalities and public goods in the presence of private information. This paper designs mechanisms that incentivize truthful revelation in nuisance, negligence, and product liability law, and enable the authority to determine optimal standards of conduct.

9 Decentralized decision making by the informed parties is an essential goal of mechanism design. It is necessary to model the initial information structure and the exchange of information necessary to coordinate the parties’ actions so that they can achieve socially desirable outcomes. Hurwicz and Reiter (2006) provide an algorithmic approach to designing minimal-information exchanges to achieve maximally decentralized mechanisms that can achieve the given social objective. Coasean transaction-cost analysis is unable to endogenously derive maximally decentralized legal institutions or require that they implement social objectives as equilibrium outcomes.
The “Coase theorem” is not a theorem.10 “The Problem of Social Cost” provides a series of examples that illustrate how—in the absence of transaction costs—private bargaining between two parties could arrive at efficient outcomes, and how—in the presence of transaction costs—the law can facilitate efficient outcomes by assigning rights and obligations appropriately. A theorem is statement that is proven rigorously, based on previously established statements (other theorems) and generally accepted statements (axioms) that are explicitly specified. There has been a great deal of argument about precisely what the Coase “theorem” says and the conditions under which it holds because it has no definitive statement.11 It seems to be an intellectual and ideological Rorschach “theorem” that many scholars insist means just what they choose it to mean—neither more nor less.12

Leonid Hurwicz (1995) points out in his article, “What is the Coase Theorem?,” that the first welfare theorem formally establishes Pareto efficiency of competitive equilibrium—under assumptions that exclude externalities, public goods, market power, asymmetric information, and incomplete markets.13 He argues that Coase’s definition of transaction costs as anything that prevents the parties from arriving at efficient allocations makes the “theorem” true by definition. It does not extend the efficiency results beyond the classical assumptions for competitive equilibrium.

Without a precise statement of the theorem and a proof that it holds under some set of necessary and sufficient conditions, the Coase “theorem” is insufficient to establish that appropriately delineated property rights enable private bargaining to overcome market failures (c.f., Harold Demsetz, 2011). Invoking the Coase “theorem” does not excuse economic analysts of law from these basic requirements of modern economics.

Efforts to formalize the theorem invariably show that it does not hold under a variety of important cases, including among others: bilateral monopoly negotiations in Bayesian equilibria

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10 George Stigler apparently was the first person to use the term “Coase theorem.” Coase did not state the idea as a theorem in “The Problem of Social Cost.”
11 Coase responded to criticisms that his work lacked mathematical precision saying, “In my youth it was said, that which is too silly to say may be sung. In modern economics, that which is too silly to say is put into mathematics.” (Coase, 1981).
12 “When I use a word,” Humpty Dumpty said, in a rather scornful tone, “it means just what I choose it to mean—neither more nor less.”
“The question is,” said Alice, “whether you can make words mean so many different things.”
“The question is,” said Humpty Dumpty, “which is to be master—that’s all.”? (Lewis Carroll, Through the Looking Glass.)
13 Existence of a competitive equilibrium and the welfare theorems also depend on precise and well known mathematical assumptions (see Arrow and Debreu (1954) and Debreu (1959).
when people require payment in return for revealing private information (Patrick Bolton and Mathias Dewatripont, 2005), non excludable externalities among more than two agents (Sandeep Baliga and Eric Maskin, 2003), multi-party negotiations with no core allocations (Varouj Aivazian and Jeffery Callen, 1981), continuous renegotiation of a game’s payoffs between two parties (Matthew Jackson and Simon Wilkie, 2005), and infeasibility of voluntary and efficient provision of public and private goods in Bayesian equilibrium among individuals with private information (Jackson, 2003).

The central Coasean concept of “transaction costs” is ill defined. The literature does not have a systematic approach to modeling transaction costs as arising endogenously from the legal and economic environment that produces the legal dispute. Coase provides examples of ad hoc transaction costs, including bargaining time, legal fees, court costs, consulting fees, and costs of monitoring and verification. When Coaseans do not model transaction costs as arising endogenously from the economic environment, their exogenous choice of alternative legal regimes cannot mitigate transaction costs endogenously as equilibrium outcomes of their models.

The concept of transaction costs should be generalized as “impediments to private negotiation” arising endogenously from private information, strategic behavior, instability of coalitions between parties, incentive constraints, inconsistency of voluntary participation over time, problems of endless renegotiations of the terms of agreements, and private budget constraints that prevent implementation of socially optimal outcomes. When these impediments to bargaining arise endogenously, mechanism design can endogenously determine the rules and institutions that implement social goals as equilibrium outcomes.

The social goal in the Coasean literature is maximization of wealth, profit, consumer and/or producer surplus. The law and economics literature often claims that common law judicial decisions, which implicitly assume full information, tend towards efficiency (see e.g., Paul Rubin, 1977 and Richard Posner, 2011). Such goals are often consistent with common law doctrines, and (surprisingly) with our moral intuition, but not inevitably so (Posner, 1983).

The law also seeks to implement other social objectives. For example, compensation promotes “justice,” not efficiency. Compensation is neither necessary nor sufficient for efficiency in nuisance cases with private information, as shown below. Parties require informational rents to induce them to reveal private information. Compensation often distorts the incentives for truth revelation that is necessary to determine efficient activity levels. Likewise in torts, the Coasean literature often asserts that compensatory damages in negligence are consistent with minimizing
the cost of accidents (see e.g., Guido Calabresi, 1970). When the costs of precaution or damages levels are private information, however, this is not usually true because of informational rents.

Similarly, contract doctrines often promote “fairness,” not efficiency. A long line of economic analyses of law asserts that the purpose of contractual default provisions is to mitigate inefficiencies caused by incomplete contracts (see e.g., Shavell, 1984). Yet the economics of contracts literature (which largely developed separately from economic analysis of law) demonstrates the irrelevance of contractual incompleteness for joint wealth maximization among contracting parties (see e.g., Tirole, 1999, Schwartz, 2003). Since there are mechanisms that can achieve efficiency in incomplete contracts, these default provisions require some other justification—such as fairness.

4. Paradigm change

More than fifty years after publication of “The Problems of Social Cost,” the Coasean paradigm still dominates economic analysis of the law. The literature often appeals to the Coase “theorem” to claim that private bargaining will necessarily solve externality problems (Demsetz, 2011), and will usually solve public goods problems (see e.g., Coase, 1974 and Carl Dahlman, 1979), without requiring either a well-specified generally applicable theorem, or a clear statement and proof for specific cases. Coaseans often argue that government failure is more problematic than market failure because an authority supposedly cannot obtain private information on costs and benefits that is required to optimize outcomes (see e.g., Coase 1960, and James Buchanan 1999). Many economists accept these arguments uncritically, without basing them on modern economic models with private information, strategic behavior, incentive constraints, and truth revelation in equilibria that implement societal goals. Economists teach the Coase “theorem” to their students, to lawyers, to judges, and to other policy makers. This message has permeated the public consciousness in the form of market failure denial.

The Coasean approach to law and economics, although inadequately founded in modern microeconomic theory, will not be displaced until a more powerful, inclusive, and well-founded paradigm is available. This paper is not primarily intended to argue the shortcomings of the Coasean approach, but rather to demonstrate that mechanism design provides such a paradigm. Mechanism design can replicate Pigouvian and Coasean results as special cases, and shows these results often do not extend to more general and rigorous environments with endogenous transaction costs and equilibrium outcomes.

The theory of mechanism design has produced “classical” mechanisms with necessary and

Economic analysis of law should adapt these “classical” mechanisms and their implementation theorems from the existing literature to implement legal objectives as equilibrium outcomes of strategic interactions among privately informed agents. It should also employ the applied mechanism design techniques of Rakesh Vohra (2011) and Leonid Hurwicz and Stanley Reiter (2006) to create new mechanisms that model laws as endogenously designed incentive systems. Mechanism design provides a new, richer paradigm for economic analysis of law by integrating the economics of information, game theory, incentive compatibility, and social choice.14

5. Designing new mechanisms

Now I design two new liability mechanisms to illustrate the application of mechanism design to law and economics. The technique of Vohra (2011) implements the social objective of efficiency in dominant-strategy or Bayesian equilibrium for agents with quasi-linear preferences that satisfy the no-negative-cycle condition.15 The technique of Hurwicz and Reiter (2006) applies to more general social choice correspondences, individual preferences, and equilibrium concepts, subject to classical implementability theorems. I apply these approaches to Coase’s farmer-rancher example from “The Problem of Social Cost” and to Hand’s negligence and product

14 Other authors have applied mechanism design to problems in law and economics, but it has yet to become a systematic approach to the field. (See e.g., Hurwicz (1995), Baliga and Maskin, (2003), Tirole (1999), Bolton and Dewatripont (2005), Jackson (2003), Diamantes (2009), and Salanie (2005).

15 The no-negative-cycle condition is a network version of a monotonicity requirement that is necessary and sufficient for implementability in dominant or Bayesian strategy equilibrium, see Vohra (2011) p 72 for details.
liability tests from his T. J. HOOPER (1932) and CARROLL TOWING (1947) opinions.

5.1 Applied mechanism design with allocation networks

Vohra’s technique represents mechanisms as networks of nodes and arcs, which correspond to allocations and incentive constraints (including participation conditions). Agents report their valuations of alternative allocations and choose among allocation nodes (which constitute their messages and strategies), subject to prices (aka, tax-transfers or liability) associated with the selection of each allocation node, as determined by the outcome rules. These prices depend on the reported types of other players and an initial allocation node, which represents the initial delineation of legal rights. The allocation prices are the least cost-path through the allocation network from the initial node to the agents’ chosen node. The cost of traversing each arc along the least-cost path is the decrease in expected value to the other agents at the arc’s destination node relative to its origin node, as reported by the other agents. Each agent’s choice of node is equivalent to claiming that its allocation is the best for his type, subject to the other agents’ reports and the rule (liability) functions. A simple algorithm developed in Vohra (2011), and further elaborated here, determines the allocation nodes and their prices. The network representation of mechanisms corresponds directly to linear programming, but in many cases, all that is necessary to determine the optimal liability rules is a simple algorithm using the network graphs and arithmetic (see, Joseph Daniel, 2015, for numerous examples).

To apply Vorha’s technique, let $i \in I \subset \mathbb{N}$ represent the agents, which have types $\theta_i \in \Theta_i$. We restrict the allocations $z \in Z$ to those for which some agent $i$ of type $\theta_i$ has distinct valuations $V_i: Z \rightarrow \mathbb{R}$, and which are optimal under the social objective $F: \Theta \rightarrow Z$ for some type profile $\theta \in \prod_{i \in I} \Theta_i = \Theta$. Thus, $Z = \{z \mid [z \neq z \Rightarrow V_i(z_i|\theta_i) \neq V_i(z_i'|\theta_i)], \text{ for some } i \in I\} \bigwedge [z = F(\theta_i, \theta_{-i}), \text{ for some } \theta_i \in \Theta_i]\}$. To write the liability functions in terms of the allocations instead of the types, we use the taxation principle:

If $F$ is implementable in dominant strategies (IDS), then there exists liability function $\tau: \Theta \rightarrow \mathbb{R}$, such that $F(\theta) \in \arg\max_{z \in Z} \{V_i(z_i|\theta_i) - \tau_i(\theta_i)\}, \forall i \in I$.

Note that whenever $F(\bar{\theta}) = F(\bar{\theta})$ then $t_i(\bar{\theta}_i) = t_i(\bar{\theta}_i)$, so that $\tau^* \rightarrow \mathbb{R}$, can be expressed as a function of $z$.

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16 In the Coasean farmer-rancher example, the farmer will only ever choose to farm two fields, so we can ignore the farmer’s allocation network. I will add the farmer’s allocation network later for the more general bilateral externality case.
An allocation network has nodes \( z \in \mathcal{Z} \) and arc lengths given by

\[
\ell(\mathcal{Z}, \mathcal{Z}) = \inf_{\theta \in \mathcal{W}} \{ V(\mathcal{Z}|\bar{\theta}_i) - V(\mathcal{Z}|\bar{\theta}_i) \mid V(F(\theta, \theta_{-i})) = V(\mathcal{Z}) \}.
\]

This network is denoted by \( \tilde{\mathcal{Z}}_F \).

The expression for the arc length says to find the smallest difference \(^\text{17}\) between the valuations of the arc’s end points \( V(\mathcal{Z}|\bar{\theta}_i) - V(\mathcal{Z}|\bar{\theta}_i) \), over all types that are consistent with the optimality of the allocation of the destination node, \( \bar{\theta}_i \in \{ V(F(\theta, \theta_{-i})) = V(\mathcal{Z}) \} \).

Vohra (2011) presents a corollary of Rochet’s theorem (Jean-Charles Rochet, 1987) applicable to allocation networks that specifies when social choice functions are implementable in dominant strategies. Suppose there are two or more agents with type spaces \( \Theta \), quasi-linear utilities over a set of outcomes \( \mathcal{Z} \), and an allocation rule \( F: \Theta^n \rightarrow \mathcal{Z} \), then \( F \) is IDS if and only if for all agents \( i \) and reports \( \theta_{-i} \), the corresponding networks \( \tilde{\mathcal{Z}}_F \) have no finite cycles of negative length.

The condition that there are no-finite cycles of negative length can often be verified simply by inspecting the network arcs. This condition assures that the mechanism implements the social objective in dominant strategies.

Finally, we construct the liability functions by finding the least cost path from an initial node, which represents the original allocation of rights and obligations, to each of the allocation nodes,

\[
(\tau_{z_0}^\mathcal{Z}(z, z_0) = \min_{p \in \tilde{\mathcal{Z}}_F} \{ \sum_{j=0}^{\mathcal{Z}} \ell(z_j, z_{j+1}) \},
\]

where \( p = \{ z_0, z_1, ..., z \} \in \tilde{\mathcal{Z}}_F \), is a network \( \tilde{\mathcal{Z}}_F \) path from \( z_0 \) to \( z \).

For networks representing the mechanisms of law and economics, the minimum length path length can often be determined simply by inspecting the network arcs. More generally, many software packages can calculate minimal length paths for finite networks of reasonable size.

5.2 The farmer-rancher mechanism with observable external harms

Now let us derive a mechanism for a case involving agent \( r \) choosing activity levels \( x \) or \( m \in \{1, 2, ..., M\} \), where agent \( r \)’s activity has publically observable and costlessly verifiable external effects \( -h_{mn}^f \) on agent \( f \)’s output. Assume for now, as Coase, that agent \( f \)’s activity level

\(^{17}\) More correctly, the “inf” is the greatest lower bound of this difference. This means that for the purposes of this calculation, we may safely treat any strict inequality constraints on the types as if they were weak inequality constraints. The examples will help clarify.
\( y \) or \( n \in \{1, 2, \ldots, N\} \) has no effect on agent \( r \)’s output. Let \( \theta^r_m \) be the value for agent \( r \) of performing activity level \( m \). The valuation \( \theta^r_m \) is private information.

For given activity levels \( m \) and \( n \), the value of the allocations are \((\theta^r_m, \theta^r_n - h^f_{mn})\). The condition for activity level \((x, y)\) to be socially optimal is that the sum of the agents’ valuations of activity level \((x, y)\) are greater than that of all other activity levels \((m, n)\),

\[
\theta^r_x + \theta^f_y - h^f_{xy} \geq \theta^r_m + \theta^f_n - h^f_{mn}, \quad \forall \ m \in \{1, 2, \ldots, M\} \land n \in \{1, 2, \ldots, N\}.
\]

Now let \( V(a|s) \) and \( V(b|s) \) denote the utility that an agent of type \( s^i \in \Theta^i \) obtains when the joint activity level is \( \alpha = (m, n) \) or \( \beta = (x, y) \in \{1, 2, \ldots, M\} \times \{1, 2, \ldots, N\} \). The general expression for an arc length \( \ell(\alpha, \beta) \) is the infimum of \( V(b|s) - V(a|s) \) over \( s \in R^\beta \equiv \{\theta | V(F(\theta, \theta_{-i})) = V(\beta)\} \). In this example, the expression \( V(b|s) - V(a|s) \) for an agent of type \( s \in \Theta^r \) is simply \((s^r_x - s^r_m)\). The restriction that \( s \in R^\beta \) requires that \( s \) is among the types for which activity level \( \beta = (x, y) \) is socially optimal, that is

\[
s^r_x + \theta^f_x - h^f_{xy} \geq s^r_m + \theta^f_n - h^f_{mn}, \quad \forall \ m \in \{1, 2, \ldots, M\} \land y \in \{1, 2, \ldots, N\}.
\]

Cancel and rearrange so that the left hand side matches the expression for \( V(b|s) - V(a|s) \),

\[
s^r_x - s^r_m \geq h^f_{xy} - h^f_{my}, \quad \forall \ m \in \{1, 2, \ldots, M\} \land y \in \{1, 2, \ldots, N\}.
\]

It follows that the length of the arcs in \( r \)’s allocation network are:

\[
\ell^r(\alpha, \beta) = \inf_{s^i \in R^\beta} [s^r_x - (s^r_m)] = h^f_{xy} - h^f_{my}.
\]

Applying this model to the farmer-rancher problem, gives the rancher’s allocation network shown in Figure 1. The nodes represent the number of cattle and the labels on the arcs connecting the nodes indicating their lengths from Equation 6. In this network, all cycles have total length zero, and all paths between any two given nodes have the same total payments, so they are all shortest paths. We can represent different property rights and liability rules (delineation of rights) by choosing the appropriate initial node. If we want the rancher to be liable for all damage to the farmer’s crops, then we calculate the rancher’s liability starting from the node where his activity level is zero. If we want the rancher to be entitled to perform activity level three (raise three head of cattle), then we calculate the liability by starting at node three. More generally, if the initial rights are at activity level \((u, v)\), then expression for agent \( r \)’s liability for choosing allocation \((x, y)\) is

\[
\tau^r_{xy} = h^f_{xy} - h^f_{uv}.
\]
Proposition 1: The farmer-rancher mechanism with observable external harms implements the efficient activity levels in dominant strategy equilibrium. (See Appendix for proof.)

5.3 Coase’s numerical example

Coase’s well-known farmer-rancher example illustrates the power of mechanism design relative to the Coasean paradigm. The full information mechanism results in games and liability rules that reproduce Coase’s original examples as games in which the rancher chooses how many cattle to raise, and the farmer decides how many fields to farm.\(^\text{18}\) The authority in this mechanism could be an administrative agency, a legislature, or a common-law judge deciding on a liability

\(^{18}\) I extend Coase’s example to include a price of $21 per head of cattle and a cost function of \(C=H^2\), where \(H\) is the number of cattle. The farmer chooses how many fields to plant, with each field costing $10 to plant and the first field generating $28 in revenues while the second field produces $12 additional output and the third produces $9. These parameters are chosen to be consistent with Coase’s example. I will subsequently develop a general model of bilateral externalities with private information.
rule. In the farmer-rancher example, the social objective is to implement a joint-profit-maximizing outcome. Coase assumes that the revenues and costs of the rancher and farmer are common knowledge among the parties and the authority. Since the farmer’s activity level in this example has no effect on the rancher’s utility, the farmer’s liabilities are all zero, and she chooses the number of fields that produce the highest utility for her. Table 1 lists the liabilities for the rancher. Note that they are the same as the rancher’s liability levels in the Coase’s article, for the given initial property rights.

Table 1—Rancher’s Liabilities for alternative activity levels

<table>
<thead>
<tr>
<th>Head of Cattle (Node)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Property rights set at (0, 2)</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>Property rights set at (2, 2)</td>
<td>-3</td>
<td>-2</td>
<td>0</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>Property rights set at (3, 2)</td>
<td>-6</td>
<td>-5</td>
<td>-3</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

When the external harm associated with different activity levels is publically known, either the authority or the farmer could receive or pay the rancher’s payments or subsidies without upsetting the incentives for the parties to adopt the efficient activity levels, because the external costs are public information so that misrepresentation is not possible. This mechanism may be interpreted as a Pigouvian tax-transfer program, or as a nuisance compensation scheme. Under this full information structure, we have derived liability rules for addressing externality problems as the solution to a mechanism design problem, and this system is similar to the Pigouvian tax-transfers, the Coasean outcomes, and the existing property rights in nuisance cases.

The normal form game in Table 2 shows the parties’ payoffs in the form of \( \text{revenues} - \text{costs} = \text{profits} \), prior to the parties engaging in any bargaining, or a social planner or judge imposing any taxes or liability. Note that the joint-profit maximizing outcome is the dark-shaded cell, with the rancher raising two head of cattle and the farmer planting two fields. The rancher’s individually best output level, however, is three head of cattle.

Table 2—The Farmer-Rancher Game (before liability)

<table>
<thead>
<tr>
<th>Farm</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28-10=18</td>
<td>40-20=20</td>
<td>49-30=19</td>
</tr>
<tr>
<td>2</td>
<td>21-1=20, 27-10=17</td>
<td>39-20=19</td>
<td>48-30=18</td>
</tr>
<tr>
<td>3</td>
<td>63-27=36, 22-10=12</td>
<td>34-20=14</td>
<td>43-30=13</td>
</tr>
<tr>
<td>4</td>
<td>84-64=20, 18-10=8</td>
<td>30-20=10</td>
<td>39-30=9</td>
</tr>
</tbody>
</table>
A game that implements the Pigouvian approach to addressing the externality issue changes the players’ payoffs by imposing a tax on the rancher equal to the external harm he imposes on the farmer. The following game shows the after-tax payoffs of the parties in the form of $profit \pm tax = payoff$.

### Table 3—The Farmer-Rancher Game (Pigouvian taxes)

<table>
<thead>
<tr>
<th></th>
<th>1 Field</th>
<th>2 Fields</th>
<th>3 Fields</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>R</strong></td>
<td>0, 18</td>
<td>0, 20</td>
<td>0, 19</td>
</tr>
<tr>
<td><strong>a</strong></td>
<td>20-1=19, 17</td>
<td>20-1=19, 19</td>
<td>20-1=19, 18</td>
</tr>
<tr>
<td><strong>n</strong></td>
<td>34-3=31, 15</td>
<td>34-3=31, 17</td>
<td>34-3=31, 16</td>
</tr>
<tr>
<td><strong>c</strong></td>
<td>36-6=30, 12</td>
<td>36-6=30, 14</td>
<td>36-6=30, 13</td>
</tr>
<tr>
<td><strong>h</strong></td>
<td>20-10=10, 8</td>
<td>20-10=10, 10</td>
<td>20-10=10, 9</td>
</tr>
</tbody>
</table>

Note that the best strategies are for the rancher to raise two head of cattle regardless of what the farmer does, and for the farmer to plant two fields regardless of what the rancher does. The Pigouvian taxes create incentives for the parties to choose the socially desired outcome. It implements joint profit maximization as a dominant strategy equilibrium.

Now consider nuisance liability that gives the farmer the right to be free from crop damage and makes the rancher liable to compensate the farmer for damages to his crops. The payoffs are in the form of $profit \pm liability = payoff$.

### Table 4—The Farmer-Rancher Game (Rancher liability)

<table>
<thead>
<tr>
<th></th>
<th>1 Field</th>
<th>2 Fields</th>
<th>3 Fields</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>R</strong></td>
<td>0, 18</td>
<td>0, 20</td>
<td>0, 19</td>
</tr>
<tr>
<td><strong>a</strong></td>
<td>20-1=19, 17+1=18</td>
<td>20-1=19, 19+1=20</td>
<td>20-1=19, 18+1=19</td>
</tr>
<tr>
<td><strong>n</strong></td>
<td>34-3=31, 15+3=18</td>
<td>34-3=31, 17+3=20</td>
<td>34-3=31, 16+3=19</td>
</tr>
<tr>
<td><strong>c</strong></td>
<td>36-6=30, 12+6=18</td>
<td>36-6=30, 14+6=20</td>
<td>36-6=30, 13+6=19</td>
</tr>
<tr>
<td><strong>h</strong></td>
<td>20-10=11, 8+10=18</td>
<td>20-10=10, 10+10=20</td>
<td>20-10=11, 9+10=19</td>
</tr>
</tbody>
</table>

As in the case of the Pigouvian tax, nuisance liability changes the payoffs to implement joint profit maximization as a dominant strategy equilibrium. The difference is that nuisance liability results in compensation to the farmer.

Coase argues, however, that in the absence of transaction costs, the parties could also bargain to the optimal outcome by having the farmer pay the rancher an amount equal to the harm the farmer avoids by reducing the numbers of cattle. The following game shows the payments in the form of $profit \pm payment = payoff$. 

---

16
Table 5—The Farmer-Rancher Game (Farmer liability)

<table>
<thead>
<tr>
<th>$\Gamma$</th>
<th>Field 1</th>
<th>2 Fields</th>
<th>3 Fields</th>
</tr>
</thead>
<tbody>
<tr>
<td>R 0</td>
<td>6, 18-6=12</td>
<td>6, 20-6=14</td>
<td>6, 19-6=13</td>
</tr>
<tr>
<td>a 1</td>
<td>20+5=25, 17-5=12</td>
<td>20+5=25, 19-5=14</td>
<td>20+5=25, 18-5=13</td>
</tr>
<tr>
<td>n 2</td>
<td>34+3=37, 15-3=12</td>
<td>34+3=37, 17-3=14</td>
<td>34+3=37, 16-3=13</td>
</tr>
<tr>
<td>c 3</td>
<td>36, 12</td>
<td>36, 14</td>
<td>36, 13</td>
</tr>
<tr>
<td>h 4</td>
<td>20+4=24, 8+4=12</td>
<td>20, 10+4=14</td>
<td>20, 9+4=13</td>
</tr>
</tbody>
</table>

Once again, this game implements the socially desired, joint-profit-maximizing outcome as a dominant-strategy equilibrium. A court could enforce a farmer liability rule based on these payments that would also implement the joint profit-maximizing outcome. Thus the allocation of rights in this example does not affect the final distribution of real resources.

In the event of positive transaction costs that prevent the parties from bargaining to the joint profit-maximizing outcome, Coase argues for adopting the rule that maximizes the difference between all benefits and costs associated with the activities, including the cost of the authority’s intervention. The idea that the authority should choose the assignment of rights that produces the social wealth-maximizing outcome has come to be known as the “normative Coase theorem” because it provides guidance as to what the law “should” be. The mechanism design framework takes (normative) social goals exogenously and uses the (positive) concept of implementation to endogenously design legal rules that achieve the social goal as an equilibrium outcome.

The farmer-rancher mechanism with perfect information about external harms has the nice property that all we need to do is define the property rights (the initial node from which to calculate node prices) and the parties are able to arrive efficient outcomes. Many people seem believe that the Coase “theorem” guarantees such efficient outcomes. Using mechanism design to model transaction costs arising from private information about levels of harm, however, shows that this Coasean mechanism—which is compensatory—does not provide the correct incentives for truthful revelation of private costs. Such information is necessary to set the liabilities and determine efficient activity levels when costs, benefits, and external harms are private information.

5.4 The bilateral farmer-rancher problem with private information on net benefits of activities

Now suppose that the benefits and costs, including the external cost that the party’s activity levels impose on one another, are private information known only to the agent that experiences

\[\text{19 See, Cooter and Ulen (2012).}\]
them. The authority has to rely on the agents to report the change in net benefit they experience as a result of changes in activity levels. We design a mechanism for a general the case where agent \( I \) chooses activity levels \( x \) or \( m \in \{1, 2, \ldots, M\} \) and agent \( j \) chooses activity levels \( y \) or \( n \in \{1, 2, \ldots, N\} \). Their net benefits depend on both activity levels and consist of the benefits of the activity less its direct cost and external cost. The agents’ have multidimensional types, \( \theta^i_{xy} \) and \( \theta^j_{xy} \in \mathbb{R}^{M \times N} \), representing their net benefits from each activity level. We construct an allocation network with nodes representing every combination of activity levels. Allocation \((x, y)\) has valuations \((\theta^i_{xy}, \theta^j_{xy})\), social value \( \theta^i_{xy} + \theta^j_{xy} \), and optimality conditions that allocation \((x, y)\) has higher social value then all other allocations

\[
\theta^i_{xy} + \theta^j_{xy} \geq \theta^i_{mn} + \theta^j_{mn}, \quad \forall \ m \in \{1, 2, \ldots, M\} \text{ and } n \in \{1, 2, \ldots, N\}.
\]

The general expression for \( V(\beta|s) - V(\alpha|s) \) in this example is simply \((s^i_{\beta} - s^i_{\alpha})\), where \( \alpha \) and \( \beta \in \{1, 2, \ldots, M\} \times \{1, 2, \ldots, N\} \). We need to determine \( \inf_{s \in R_{\beta}} [s^i_{\beta} - s^i_{\alpha}] \). The constraint that \( s \in R_{\beta} \) requires \( s \) to be among the types for which activity level \( \beta \) is socially optimal,

\[
\theta^i_{\beta} + \theta^j_{\beta} \geq \theta^i_{\alpha} + \theta^j_{\alpha}, \quad \forall \ \alpha \in \{1, 2, \ldots, M\} \times \{1, 2, \ldots, N\}.
\]

Rearranging this condition so that the left hand side matches the expression for \( V(\beta|s) - V(\alpha|s) \) yields

\[
\ell(\alpha, \beta) = \inf_{s \in R_{\beta}} [s^i_{\beta} - s^i_{\alpha}] = \theta^j_{\alpha} - \theta^j_{\beta}.
\]

In words, the arc length from nodes \( \alpha \) to \( \beta \) equals the difference in the other agent’s reported net benefits between activity levels \( \alpha \) and \( \beta \). The liability is the length of the shortest path between the initial node and the terminal node. In this model, the choice of an initial node represents an assignment of property rights allowing the agents to engage in the initial activity level without any liability. The liability rules create an incentive system that elicits truthful reporting of the agents’ private information about costs and benefits as a dominant-strategy equilibrium. Agents can do no better than to truthfully report the net benefits that they experience. As in the Coasean system, some property rights regimes tax agents for choosing activity levels that increase externalities, and others subsidize agents for choosing activity levels that reduce externalities. Unlike the Coasean system, the incentive system requires that the payments \textbf{not} be made to the other party.

The expression for arc lengths implies that in this network, all cycles have total length zero, thus satisfying no-negative cycle condition, which is sufficient for implementability in dominant strategies. All paths between any two given nodes have the same total payments, so they are all
shortest paths. We can represent different property rights and liability levels (delineation of rights) by choosing the appropriate initial node. As we traverse a path from an initial node to a destination node in order of activity levels, all the intermediate arc lengths cancel out, so the shortest length path is the difference between the other agent’s valuation of the destination and origination allocations. When the initial rights are at activity level \((u, v)\), then expression for agent \(i\)’s liabilities for choosing allocation \((m, n)\) is

\[
\tau_{mn}^i = \theta_{mn}^i - \theta_{uw}^i.
\]

The formula for the general normal form game provides payoffs equal to the agents’ true net benefits for the chosen activity level less the payments (subsidies) depending on the initial property rights set by the authority and the activity levels that the agents choose. The outcome or rule functions produce the following payoffs in a normal form game for players \(i\) and \(j\) with realized types \(\theta^i\) and \(\theta^j\in\mathbb{R}^{M\times N}\) when \(i\) chooses activity level \(m\) and \(j\) chooses activity level \(n\).

\[
(\theta_{mn}^i - \tau_{mn}^i, \theta_{mn}^j - \tau_{mn}^j)
\]

**Proposition 2:** The incompatible activities mechanism implements the efficient activity levels in dominant strategy equilibrium. (See Appendix for proof.)

Applying this general model to the farmer-rancher problem, gives the allocation network in Figure 2. The rancher chooses the number of cattle and the farmer chooses the number of fields, so there are three sub networks for the rancher with black arcs connecting the nodes that the rancher is able to choose among. There are actually five sub-networks for the farmer (for clarity, only one is shown) with gray arcs connecting the nodes that the farmer is able to choose among. Each node of the rancher’s sub networks connects to the other nodes for which the number of fields is fixed at 1, 2 or 3. The rancher cannot choose between the nodes in different sub networks, so there are no arcs (or arcs of infinite length) between these nodes. Each node of the farmer’s sub networks connects to the other nodes for which the number of cattle is fixed at 0, 1, 2, 3, or 4. The farmer is unable to choose between nodes in different sub networks, so there are no arcs connecting them.

The rancher’s sub network 1 (one field) shows the numeric value of the arcs, which (in this example, but not more generally) are the same in all his other sub networks. The arcs labels in the rancher’s sub network 2 (two fields) indicate the general expression for their lengths from Equation 15. The third sub network indicates the liabilities when the farmer and rancher have property rights to set at the activity levels (2, 3).

The following tables illustrate the application of this approach to Coase’s farmer-rancher
example. Each column gives the payment (positive) or subsidy (negative) when the initial property rights are set to the activity level given by the column header and the agents’ activity levels are given by the row header. Notice that the rancher’s payment schedule when he has no rights to pollute (lightly highlighted) is the same as in the Coase article. So too, is the rancher’s payment schedule when he has the right to raise three cattle (moderately highlighted).

The rancher’s payment schedule when he has the right to raise two cattle imposes no liability at the optimal activity level. If the rancher unilaterally deviates from the activity level permitted by his property rights, he pays the difference between the farmer’s net benefits at the rancher’s permitted and chosen activity levels. If the property rights are not initially set at the optimal activity levels, the rancher pays the difference between the farmer’s net benefits at the optimal activity level that is chosen and at the activity level for which he has property rights (bold
The farmer’s payment schedule, as a function of his choice of number of fields, is constant at zero when the rancher’s activity level is consistent with the property rights (highlighted). On the other hand, if the property rights deviate from the optimal activity levels, the farmer pays the difference between the rancher’s net benefits at the optimal activity level that is chosen and at the property-rights activity level (bold italicized font).

Suppose the authority sets the property rights at the socially optimal activity levels (two cattle, two fields), then the payments and the normal form game are as shown below. The agents reveal their true net benefit levels, will choose the optimal levels as a dominant-strategy equilibrium, and will pay no taxes or receive any subsidies. Any deviation from the activity levels that appear optimal from the agents’ reports will indicate an out-of-equilibrium false report by the deviating agent.

This property rights and liability system implements the socially efficient activity levels under general situations with private cost-benefit information, discrete activity levels, and bilateral externalities. It does not generally compensate agents for the other agent’s deviation from their property levels or have balanced budgets, except where the authority sets the property rights at the efficient level. If the property rights were previously determined (at non-optimal levels), the authority would need to maintain a fund that would receive and disburse...
payments or impose additional liabilities that are constant (lump sum) by agent. A net surplus or deficit for the agents could affect the cases submitted for resolution by such a mechanisms. It would not, therefore, be appropriate in all cases, without additional modifications. Pairing the system with compensatory insurance and imposing lump sum “court costs” could address some of these issues.

The point of this example is not that this mechanism is a perfect solution to the problem that private costs and benefits pose for all nuisance (or other externality) cases, but that mechanism design, by specifying the social objective, modeling the informational issues, deriving the rule of law as a mechanism, and requiring the solution to result from an equilibrium interaction between
### Table 8—The general normal form game from the mechanism

<table>
<thead>
<tr>
<th>Cattle</th>
<th>Fields 1</th>
<th>Fields 2</th>
<th>Fields 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>((\theta_{01} - \tau_{01}, \phi_{01} - \tau_{01}))</td>
<td>((\theta_{02} - \tau_{02}, \phi_{02} - \tau_{02}))</td>
<td>((\theta_{03} - \tau_{03}, \phi_{03} - \tau_{03}))</td>
</tr>
<tr>
<td>1</td>
<td>((\theta_{11} - \tau_{11}, \phi_{11} - \tau_{11}))</td>
<td>((\theta_{12} - \tau_{12}, \phi_{12} - \tau_{12}))</td>
<td>((\theta_{13} - \tau_{13}, \phi_{13} - \tau_{13}))</td>
</tr>
<tr>
<td>2</td>
<td>((\theta_{21} - \tau_{21}, \phi_{21} - \tau_{21}))</td>
<td>((\theta_{22} - \tau_{22}, \phi_{22} - \tau_{22}))</td>
<td>((\theta_{23} - \tau_{23}, \phi_{23} - \tau_{23}))</td>
</tr>
<tr>
<td>3</td>
<td>((\theta_{31} - \tau_{31}, \phi_{31} - \tau_{31}))</td>
<td>((\theta_{32} - \tau_{32}, \phi_{32} - \tau_{32}))</td>
<td>((\theta_{33} - \tau_{33}, \phi_{33} - \tau_{33}))</td>
</tr>
<tr>
<td>4</td>
<td>((\theta_{41} - \tau_{41}, \phi_{41} - \tau_{41}))</td>
<td>((\theta_{42} - \tau_{42}, \phi_{42} - \tau_{42}))</td>
<td>((\theta_{43} - \tau_{43}, \phi_{43} - \tau_{43}))</td>
</tr>
</tbody>
</table>

### Table 9—The liability functions and the resulting normal form game

<table>
<thead>
<tr>
<th>Liabilities by activity levels assuming rights to raise two cattle and farm two fields</th>
<th>Normal form game assuming rights to raise two cattle and farm two fields</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cattle</strong></td>
<td><strong>Fields</strong></td>
</tr>
<tr>
<td>0</td>
<td>1 (0, 14)</td>
</tr>
<tr>
<td>1</td>
<td>2 (2, 0)</td>
</tr>
<tr>
<td>2</td>
<td>3 (31, 14)</td>
</tr>
<tr>
<td>3</td>
<td>4 (13, -4)</td>
</tr>
<tr>
<td>4</td>
<td>2 (32, 15)</td>
</tr>
</tbody>
</table>

the parties, provides a better framework for law and economics than the current paradigm with its *ad hoc* specification of “transaction costs,” its lack of budgetary considerations, nonstrategic negotiating behavior, absence of incentive or participation constraints, informal “equilibrium” concepts, and its unproven (and often erroneous) claim that private bargaining will necessarily overcome problems of private information.

### 5.5 Applications to negligence and products liability law

Learned Hand is the favorite judge of many lawyer-economists. His judicial philosophy and opinions demonstrate his remarkable economic intuition that was at least half a century ahead of his time. In *United States v Carroll Towing*, he enunciated the original “Hand test” for negligence,

[T]o provide against … injuries is a function of three variables: (1) The probability [of injury] … , (2) the gravity of the resulting injury, … [and] (3) the burden of adequate precautions. Possibly it serves to bring this notion into relief to state it in algebraic terms: if the probability be called \(P\), the injury, \(L\), and the burden, \(B\), liability depends upon

---

20 In addition to the cases discussed here, he is best known for his opinion in *United States v. Alcoa*, 148 F.2d 416 (2d Cir. 1945), which gave rise to a large literature on market share analysis for durable goods.
whether $B$ is less than $L$ multiplied by $P$, i.e., whether $B < P L$. 159 F.2d 169 (2d Cir. 1947)

Modern economics usually restates the Hand test with the probability, loss, and burden expressed as functions of continuous levels of care $x$. Let $E[x] = P(x) L(x)$ be the expected loss, $\theta$ be the unit cost of care, and $B(x) = \theta x$ be the burden of care. The optimal level of care $x^*$ minimizes the social cost of accidents $E[x] + B(x)$, which occurs where the marginal expected loss equals the marginal burden, i.e., $E'[x] + B'(x) = 0$. Since $B'(x) = \theta$, it follows that the optimal level of care is the inverse function of the derivative of expected loss, evaluated at the unit cost of care, $x^* = [E']^{-1}(\theta)$. The Hand negligence schedule $N(x)$ equals the expected loss $E[x]$ when care levels $x$ are less than the social-cost-minimizing level of care $x^*$, and zero for care levels greater than or equal to $x^*$. Under the Hand rule, the tortfeasor’s total expected cost schedule $T_N(x)$ equals $N(x) + B(x)$. The strict liability schedule $S(x)$ equals the expected loss $E[x]$ regardless of the level of care. Under strict liability, the tortfeasor’s total expected cost schedule $T_S(x)$ equals $E(x) + B(x)$.

Figure 3 illustrates the modern Hand negligence schedule and the strict liability schedule. The graph on the left shows that when the tortfeasor is confronted with the Hand negligence standard, he minimizes his own total cost function $T_N(x)$ by choosing the socially optimal level of care $x^*$. The graph on the right illustrates the strict liability schedule $S(x)$ and the expected full cost of precaution plus the expected loss. Note that both full cost schedules reach their minima at the efficient of care, but negligence liability drops to zero at the optimal level of care, while strict liability continues along the expected accident cost curve.

The Hand negligence rule implicitly assumes that the judge has perfect information about how the expected loss and burden functions vary with the level of care, so that she can set the

![Figure 3—Negligence and strict liability schedules](image-url)
standard of care at the efficient level. The strict liability rule does not require such knowledge. If
the negligence standard is not set optimally, then the potential tortfeasor will adopt the inefficient
standard of care, unless the standard is so high that the burden of precautions exceeds the
minimum social cost. In the latter case, the potential tortfeasor will return to the efficient level of
care.

5.6 The Learned Hand Negligence Mechanism (with unilateral care and public information about
victims’ losses, private information on cost of precaution)

Using Vohra’s technique, we can design a general liability mechanism that implements the
social objective of minimizing the sum of the cost of care plus expected losses. It is similar to
either the Hand negligence rule or the strict liability rule, depending on the initial delineation of
rights to engage in potentially injurious activity. We will start with a discrete choice of care
levels, and then derive a continuous mechanism by taking limits as the difference in levels of care
goes to zero. We assume that the level of care and the relationship between levels of care and
expected losses are general knowledge, but the cost of care is not necessarily known.21

We seek a mechanism that assures an agent $T$ (the potential tortfeasor) with unit cost of
precaution $\theta \in \mathbb{R}_+$ (which may be unobservable) will choose a socially efficient level of care,
represented by an allocation node. Using the same notation as in the previous section, we
construct an allocation network with three nodes representing the allocation of costs and expected
losses associated with choice of low, medium, and high levels of care, i.e.,

$z(x_l|\theta), z(x_m|\theta), \text{ and } z(x_h|\theta)$. Before imposition of liability, agent $T$ of type $\theta$ who chooses
allocation $z(x_l|\theta)$ will experience cost of care $z_T(x_l|\theta) = -\theta x_l$ and impose expected losses of
$z_V(x_l|\theta) = -E(x_l)$ on agent $V$ (the potential victim). The social allocation is $z_S(x_l|\theta) = (z_T(x_l|\theta),
 z_V(x_l|\theta)) = (-\theta x_l, -E(x_l))$. The difference in the potential tortfeasor’s valuations
between choosing a destination node $z(x_l|\theta)$ and an origination node $z(x_j|\theta)$ when his realized
type $s$ is

$$V_T(z(x_l|s)) - V_T(z(x_j|s)) = (-s x_l) - (-s x_j) = s(x_j - x_l).$$

The conditions for the allocation node $z(x_l|\theta)$ to be socially optimal are that the value of its
social allocation is greater than the social value of all other allocations, i.e.,

$$-\theta x_l - E(x_l) \geq -\theta x_j - E(x_j), \forall x_j \in \{x_l, x_m, x_h\},$$

which may be rearranged so the left hand side matches the expression in Equation 24,

21 Other assumptions about the information structure are possible, but this specification seems most plausible.
Society observes how people act and the resulting injuries so it can estimate expected harms as a function of the
level of care. The cost of care, however, may be private information known only to the potential tortfeasor.
Now define the set
\[ R_{x|\theta} = \{ \theta; \ s.t. \ \theta(x_j - x_i) \geq -(E(x_j) - E(x_i)) \}. \]

The expression for the arc length from \( z(x_j|\theta) \) to \( z(x_i|\theta) \) is
\[ \ell \left( z(x_j|\theta), z(x_i|\theta) \right) = \inf_{s \in R_{x|\theta}} \left[ s(x_j - x_i) \right] = -(E(x_j) - E(x_i)). \]

Note that the two cycle condition,
\[ \ell \left( z(x_j|\theta), z(x_i|\theta) \right) + \ell \left( z(x_i|\theta), z(x_j|\theta) \right) = -(E(x_j) - E(x_i)) - (E(x_i) - E(x_j)) \geq 0, \]
is satisfied, which is sufficient for implementability in dominant strategies because \( Z_F \) is finite and the type space is closed and convex (see Vohra, 2011, p. 45).\(^{22}\)

Figure 4 illustrates the general liability network. The liabilities \( \tau(z(x_j)) \) are the shortest path from whichever node the authority chooses as the initial allocation \( z(x_i) \), which represents the standard level of care, to the allocation chosen by the potential tortfeasor. This network has the property that all paths between two nodes have the same length. Thus the general expression for the liabilities are
\[ \tau(z(x_j)) = E(x_j) - E(x_i). \]

To get strict liability, the authority imposes an initial price of \( \tau_s(z(x_i)) = E(x_i) \), so that the liabilities at each node are
\[ \tau_s(z(x_j)) = E(x_j). \]

In other words, the potential tortfeasor always pays for the expected loss. Since we are assuming public information about the victims’ losses, the victim cannot misrepresent her true loss. The authority may transfer the injurer’s payment to the victim. If we relax the assumption of perfect information, then the victim’s payment, if any, must be independent of her claim of loss. This situation is identical to a Pigouvian tax, which goes to the authority.

To get a liability schedule somewhat similar to negligence, the authority sets the price at some standard level of care, say medium, at which the potential tortfeasor has no liability. Then the tortfeasor pays \( \tau(z(x_i)) = E(x_i) - E(x_m) \) if he exercises low care, and receives a subsidy of \( \tau(z(x_h)) = E(x_h) - E(x_m) \) if he exercises a high level of care.

This liability schedule differs from negligence in several respects. The standard of care is a permitted level, but it is not the cost minimizing care level for the high or low types. The potential tortfeasor always chooses the optimal care level for his realized type. Liability is not based on

\(^{22}\) This is an instance of the theorem of M. Saks and L. Yu (2005).
fault. It is based on incentivizing the optimal level of precaution when the cost of care is unobservable. Assuming public information on expected losses, the payment may be used to compensate the party that loses from any deviation from the standard of care. This situation is similar to Coase’s farmer-rancher example where the rancher is compensated for the injury he refrains from imposing on the farmer when he deviates from the permitted activity level. The authority, however, does not need to observe the potential tortfeasor’s cost of care. Relaxing the assumption of observable losses, however, means that the victim’s payment, if any, must be independent of her claim of loss.

We can easily convert this discrete mechanism into continuous care mechanism. Taking the limit of the change in expected costs with respect to the level of care, we obtain

\[
\lim_{x_j - x_i \to 0} \frac{E(x_j) - E(x_i)}{x_j - x_i} = \frac{d}{dx} E(x).
\]

The liability \( \tau(z(x_j)) \) at allocation \( j \) when the standard care level is \( x_i \) is, therefore,

\[
\tau(z(x_j)) = \int_{x_i}^{x_j} \frac{d}{dx} E(x) \, dx = E(x_j) - E(x_i).
\]

The continuous mechanism has all the same properties as the discrete mechanism. In particular, if the liability at \( x = \infty \) is set at \( E(\infty) \), we get the strict liability schedule \( \tau_S(z(x_j)) = E(x_j) \). If the liability at some permitted level \( x = \bar{x} \) is set at 0, we get a “negligence” liability schedule \( \tau_N(z(x_j)) = E(x_j) - E(\bar{x}) \) that assesses liability when \( x < \bar{x} \) and subsidizes the tortfeasor’s adoption of \( x > \bar{x} \) by paying an amount equal to the reduction in the victim’s expected loss. The tortfeasor minimizes his costs by adopting the socially efficient level of care given his cost of care \( \theta \), even though the authority does not observe it. This schedule eliminates the discontinuous change in liability at the permitted care level.

5.7 Bilateral Care

In many cases both the victim and the tortfeasor can take precaution to avoid injuries. The law seeks to provide incentives for the victim to take reasonable precaution to avoid harm. The traditional legal approach to bilateral care is to impose the contributory negligence rule:
contributory negligence is a complete bar to recovery (see, BUTTERFIELD V. FORRESTER, 1809). The rule confronts the victim with a negligence liability schedule like that in Figure 3. If the victim does not meet her standard of care (which is set to minimize the sum of accident and care taking costs), then she bears her own cost of care plus her expected accident costs. Therefore, she will adopt the optimal level of care to avoid her expected accident costs. The potential tortfeasor realizes that the victim will meet the standard of care and not be barred from recovery. He also adopts the optimal level of care, therefore, under either the negligence or strict liability standards.

Forty years ago, the traditional contributory negligence rule described the law of nearly every jurisdiction in the United States. Since then, over half of the states have adopted some version of the comparative negligence rule (see, HOFFMAN V. JONES, 1973). Comparative negligence assigns liability to each party in proportion to their share of total fault. Proponents of comparative negligence usually argue that contributory negligence is unfair because small differences in the victim’s behavior can cause her to lose all compensation for serious injuries. The comparative negligence rule, however, does not incentivize optimal care taking. Neither does it consider how to elicit truthful information about costs of care and damage levels, nor does it provide guidance for determining relative levels of blameworthiness.

We can interpret bilateral care in tort as similar to mutually incompatible activities in nuisance cases and apply the same incompatible activities mechanism as designed above to negligence cases. For automobile accidents, for example, the activity levels could be the degree of offensive driving (speed, acceleration, deceleration, lane changes, tailgating) and defensive driving (attentiveness, reaction time, constancy of speed, space cushions, lack of distractions). Authorities, such as insurance companies, could assess the costs of different activity levels and price them as provided by the incompatible activities mechanism with perfect information in Section 5.3. Alternatively, drivers could self report their expected net values of different activity levels to account for different unobservable values of time, risk acceptance, levels of harm, etc. Authorities, such as insurance companies, could observe the activity levels and reported net benefit-costs, and then price according to the mechanisms in Section 5.4. Instead of revisiting these mechanisms in detail, I design a new mechanism that illustrates the Hurwicz-Reiter (H-R) method (2006) of mechanism design.

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23 Automotive black boxes already (or will soon) record many of these observations.
5.8 The Benighted Hand Mechanism (with bilateral care, private information, and shared losses and liability)

Now we develop a negligence mechanism for cases with bilateral care, different effectiveness of caretaking, shared losses, and shared liability when the cost of care is private information that is unverifiable by the authority (a benighted Hand). The authority is nevertheless charged with implementing the social-cost-minimizing levels of care. The authority observes the levels of care and knows the functional relationship between care taking and expected losses based on previous observations of care levels and resulting accident records.

There are two agents, 1 and 2, who engage in activities that produce mutual risks of harm. The agents have private types, \( \theta_i \in \Theta_i = \mathbb{R}_+ \), that represent their private costs per unit of care. Let \( \Theta = \Theta_1 \times \Theta_2 \) be the type space. The levels of care are measured in units \( x_i \in X_i = \mathbb{R}_+ \), which are publically observable. Let \( X = X_1 \times X_2 \), so the care levels \( (x_1, x_2) \in X \). Define \( L_s: X \to \mathbb{R} \) to be the expected social loss function, with individual expected loss functions \( L_i: X \to \mathbb{R} \), such that \( L_1(x_1, x_2) = \gamma L_s(x_1, x_2) \) and \( L_2(x_1, x_2) = (1 - \gamma) L_s(x_1, x_2) \) for \( \gamma \in (0, 1) \). Define the function \( F_s: X \times \Theta \to \mathbb{R} \) to be the expected social costs of accidents function, with \( F_s(x_1, x_2; \theta_1, \theta_2) = L_s(x_1, x_2) + \theta_1 x_1 + \theta_2 x_2 \) (H-R call this the criterion function). It is useful to specify the social loss with a Cobb-Douglas form, to provide an example that allows the parties’ care levels to have different relative degrees of effectiveness \( \alpha \) and \( (1 - \alpha) \), where \( \alpha \in [0, 1] \). Let the expected social loss function be

\[
L_s(x_1, x_2) = \frac{1}{x_1^\alpha x_2^{1-\alpha}},
\]

where \( x_1 \) and \( x_2 \) are the levels of care. The full social cost of accidents is

\[
F_s(x_1, x_2; \theta_1, \theta_2) = \frac{1}{x_1^\alpha x_2^{1-\alpha}} + \theta_1 x_1 + \theta_2 x_2.
\]

Differentiating with respect to \( x_1 \) and \( x_2 \) gives the first order conditions, which are also known as the equilibrium equations.

\[
\begin{align*}
-\alpha x_1^{-1-\alpha} x_2^{-1+\alpha} + \theta_1 &= 0, \quad \text{and} \\
-(1-\alpha)x_1^{-\alpha} x_2^{-2+\alpha} + \theta_2 &= 0
\end{align*}
\]

\[
\implies \frac{\theta_1}{\theta_2} = \frac{\alpha x_2}{(1-\alpha)x_1}.
\]

Solving for \( x_1 \) and \( x_2 \) in terms of one another, and substituting out \( x_2 \) and \( x_1 \) in the first order conditions gives give the social-cost-minimizing care levels as functions of the (unobservable) cost parameters (which H-R call the individual goal functions)

\[
\begin{align*}
\xi_1 &= F_1(\theta_1, \theta_2) = \left(\frac{\alpha}{\theta_1}\right)^{\frac{1}{\alpha}} \left(\frac{\alpha \theta_2}{(1-\alpha) \theta_1}\right)^{\frac{1-\alpha}{\alpha}}, \quad \text{and} \\
\xi_2 &= F_2(\theta_1, \theta_2) = \left(\frac{1-\alpha}{\theta_2}\right)^{\frac{1}{\alpha}} \left(\frac{(1-\alpha) \theta_1}{\alpha \theta_2}\right)^{\frac{\alpha}{1-\alpha}}.
\end{align*}
\]

Substituting the optimal values of \( x_1 \) and \( x_2 \) into the social loss function and simplifying, we get...
the indirect social losses as a function of the unit costs of care

\[ L_s(\theta_1, \theta_2) = \left( \frac{1}{(\theta_1^2 \alpha \theta_2)} \right) \frac{1}{(1-a) \theta_1^2} \frac{\theta_1^2}{(1-a) \theta_2^2} \]  

We seek a direct mechanism with message space \( M \) equal to the type space \( \Theta \), \( M = \theta = \mathbb{R}_+^2 \), individual equilibrium functions \( g_i: M \times \Theta^i \to X_i \) and outcome functions \( h: M \to X = \mathbb{R}_+^2 \), such that the solutions to the individual goal functions is to truthfully report \( m_i = \theta_i \), i.e., \( g_i(m_i, \theta_i) = 0 \Rightarrow (m_i - \theta_i) = 0 \), and the chosen level of care is consistent with the equilibrium equations and the goal functions, i.e., \( x_1 = F_1(\theta_1, \theta_2) = h_1(m_1, m_2) \) and \( x_2 = F_2(\theta_1, \theta_2) = h_2(m_1, m_2) \).

To determine the individual equilibrium functions, \( g_i(m_i, \theta_i) \), we modify the agents’ individual objective functions \( L_i(F(m)) + \theta_i F_i(m) \) by imposing individual liability functions \( \tau_i: M \to \mathbb{R} \), such that the derivative of the resulting individual objective function is a non zero multiple of \( (\theta_i - m_i) \). This liability function causes the individual objective to be optimized at \( m_i = \theta_i \). Define the agents’ modified objective functions \( G_i: M \times \Theta \to \mathbb{R} \), such that \( G_i(m_i, \theta_i) = L_i(F(m)) + \tau_i(m_i) + \theta_i F_i(m) \).

Let \( \gamma \) be the fraction of the loss experienced by agent 1, then the modified individual objective functions with \( x_i \) set optimally and including the liability functions are

\[ G_1(F(m), \theta_1) = \gamma \left( \frac{1-a}{\theta_2} \right)^{-\frac{1-a}{2}} \left( \frac{\theta_1}{\theta_1} \right)^{-\frac{a}{2}} + \tau_1(m) \]  

\[ G_2(F(m), \theta_2) = (1 - \gamma) \left( \frac{1-a}{\theta_2} \right)^{-\frac{1-a}{2}} \left( \frac{\theta_1}{\theta_1} \right)^{-\frac{a}{2}} + \tau_2(m) \]  

Isolate the braced terms on the left hand side, differentiate with respect to \( m_1 \) and \( m_2 \), set the derivatives to zero, and substitute \( m_i \) for \( \theta_i \) on the right hand sides.

\[ \partial_{m_1} \left[ \gamma \left( \frac{1-a}{\theta_2} \right)^{-\frac{1-a}{2}} \left( \frac{\theta_1}{\theta_1} \right)^{-\frac{a}{2}} + \tau_1(m) \right] = \frac{1+\gamma(1-a)}{2 m_1} \left( \frac{\theta_2}{\theta_2} \right)^{-\frac{a}{2}} \left( \frac{1-a}{1-a \theta_1} \right)^{\frac{a}{2}} \]  

\[ \partial_{m_2} \left[ (1 - \gamma) \left( \frac{1-a}{\theta_2} \right)^{-\frac{1-a}{2}} \left( \frac{\theta_1}{\theta_1} \right)^{-\frac{a}{2}} + \tau_2(m) \right] = \frac{1+\gamma(1-a)}{2 m_2} \left( \frac{1-a}{\theta_2} \right)^{-\frac{a}{2}} \left( \frac{1-a}{1-a \theta_1} \right)^{\frac{a}{2}} \]  

Set the terms in braces equal to the negative of integral of \( m_i \partial_{m_i} F_i(m) \), so that its derivative is \( m_i \partial_{m_i} F_i(m) \) and Equations 52 will reduce to \( (\theta_i - m_i) \partial_{m_i} F_i(m) = 0 \). Integrate both sides of the equations with respect to \( m_i \) and solve for the liability functions

\[ \tau_1(m_1; m_2, \theta) = (1 + (1-a)) \left( \frac{m_1}{\alpha} \right)^{\frac{1-a}{2}} \left( \frac{\alpha m_2}{(1-a) m_1} \right)^{\frac{1-a}{2}} - \gamma \left( \frac{1-a}{\theta_2} \right)^{-\frac{a}{2}} \left( \frac{\theta_2}{\alpha} \right)^{-\frac{a}{2}} \]  

30
\[ \tau_2(m_2; m_1, \theta) = (1 + \alpha) \left( \frac{m_2}{1 - \alpha} \right)^{\frac{1}{2}} \left( \frac{(1 - \alpha) m_2 \theta_2}{m_2 - \theta_2} \right)^{\frac{\alpha}{2}} - (1 - \gamma) \left( \frac{1 - \alpha}{\theta_2} \right)^{1 - \frac{\alpha}{2}} \left( \frac{\theta_2}{\theta_1} \right)^{\frac{\alpha}{2}}. \]

Now check that solving the modified individual objective function generates the individual equilibrium equations, which solve for \( m_1 = \theta_1 \) and \( m_2 = \theta_2 \)

\[ g_1(m; \theta_1) = \partial_{m_1} \left[ (1 + (1 - \alpha)) \left( \frac{m_1}{m_1 - \theta_1} \right)^{\frac{1}{2}} \left( \frac{1 - \alpha}{m_1 - \theta_1} \right)^{\frac{\alpha}{2}} + \theta_1 \left( \frac{m_1}{m_1 - \theta_1} \right)^{\frac{1}{2}} \left( \frac{1 - \alpha}{m_1 - \theta_1} \right)^{\frac{\alpha}{2}} \right] = \]

\[ = \frac{1 + (1 - \alpha)}{2 m_1} \left( \frac{1 - \alpha}{m_1 - \theta_1} \right)^{\frac{1}{2}} \left( \frac{1 - \alpha}{m_1 - \theta_1} \right)^{\frac{\alpha}{2}} (m_1 - \theta_1) = 0 \Rightarrow (m_1 = \theta_1), \text{ and} \]

\[ g_2(m; \theta_2) = \partial_{m_2} \left[ (1 + \alpha) \left( \frac{m_2}{1 - \alpha} \right)^{\frac{1}{2}} \left( \frac{(1 - \alpha) m_1}{m_2 - \theta_2} \right)^{\frac{\alpha}{2}} + \theta_2 \left( \frac{m_2}{m_2 - \theta_2} \right)^{\frac{1}{2}} \left( \frac{(1 - \alpha) m_1}{m_2 - \theta_2} \right)^{\frac{\alpha}{2}} \right] = \]

\[ = \frac{1 + \alpha}{2 m_2} \left( \frac{1 - \alpha}{m_2 - \theta_2} \right)^{\frac{1}{2}} \left( \frac{1 - \alpha}{m_2 - \theta_2} \right)^{\frac{\alpha}{2}} (m_2 - \theta_2) = 0 \Rightarrow (m_2 = \theta_2). \]

The second derivatives of \( G_1(F(m), \theta_1) \) and \( G_2(F(m), \theta_2) \) with respect to \( m_1 \) and \( m_2 \) are clearly positive for all admissible values of \( \alpha \), at \( m_1 = \theta_1 \), and \( m_2 = \theta_2 \), hence the second order necessary conditions for a minimum are satisfied.

Confronted with these liability functions \( \tau_1(m) \) and \( \tau_2(m) \), the agents will truthfully report \( m_1 = \theta_1 \) and \( m_2 = \theta_2 \), and they will adopt the social-cost-minimizing level of care, even though the Benighted Hand authority relies on the agents’ reports of the costs of precaution to determine the optimal activity levels.

There are several important relationships between the parties’ costs under the mechanism and their expected losses and costs of precaution. These are easy to state, but their proofs are tedious and relegated to the Appendix.

**Proposition 3:** The sum of each party’s total costs of accidents, liability, and precaution under the mechanism are equal. The mechanism causes them to share the full costs equally,

\[ \gamma L_2(\theta_1, \theta_2) + \tau_1(m_1; m_2, \theta) + \theta_1 \tilde{\gamma}_1(\theta_1, \theta_2) = \]

\[ (1 - \gamma) L_2(\theta_1, \theta_2) + \tau_2(m_2; m_1, \theta) + \theta_2 \tilde{\gamma}_2(\theta_1, \theta_2). \]

**Proposition 4:** Each party’s liability under the mechanism, paid to the authority, equals the share of the losses that the other party directly experiences plus the other party’s expenditure on precaution.

\[ \tau_1(m_1; m_2, \theta) = (1 - \gamma) L_2(\theta_1, \theta_2) + \theta_2 \tilde{\gamma}_2(\theta_1, \theta_2), \text{ and} \]

\[ \tau_2(m_2; m_1, \theta) = \gamma L_2(\theta_1, \theta_2) + \theta_1 \tilde{\gamma}_1(\theta_1, \theta_2). \]

**Proposition 5:** Each party’s total costs of accidents, liability, and precaution equal twice the social loss from accidents.
\[(P5) \quad \gamma L_s(\theta_1, \theta_2) + \tau_1 (m_1; m_2, \theta) + \theta_1 \bar{x}_1(\theta_1, \theta_2) = 2 L_s(\theta_1, \theta_2), \text{ and} \]
\[(1 - \gamma) L_s(\theta_1, \theta_2) + \tau_2 (m_2; m_1, \theta) + \theta_2 \bar{x}_2(\theta_1, \theta_2) = 2 L_s(\theta_1, \theta_2). \]

**Proposition 6:** The sum of the parties’ expenditure on precaution equals the social loss from accidents

\[(P6) \quad \theta_1 \bar{x}_1(\theta_1, \theta_2) + \theta_2 \bar{x}_2(\theta_1, \theta_2) = L_s(\theta_1, \theta_2). \]

The parties’ liabilities \( \tau_1 (m_1; m_2, \theta) \) and \( \tau_2 (m_2; m_1, \theta) \) subtract out the losses that they directly experience, so each party bears the cost of its loss and precaution directly while paying the authority the other party’s cost of its loss and precaution. The authority receives an amount equal to the social loss \( L_s(\theta_1, \theta_2) \) plus the parties’ expenditures on precaution \( \theta_1 \bar{x}_1(\theta_1, \theta_2) + \theta_2 \bar{x}_2(\theta_1, \theta_2) = L_s(\theta_1, \theta_2) \). When the authority observes the losses, as we assume here, it can compensate the parties’ losses without disrupting the incentives for them to report truthfully \( m_t = \theta_t \). It cannot, however, rebate the expenditures on precaution based on the values \( m_t \) that parties’ report as \( \theta_t \) without disrupting the incentive system. The authority could return the expected values of these amounts lump sum, based on estimates from (truthful) reports in other cases involving similar parties. Thus the mechanism could operate as a type of insurance policy.

Figure 5 illustrates how the mechanism operates as a system for efficiently allocating the expected losses and cost of care between the agents, by decomposing the costs that the agents face under the mechanism. The vertical axis indicates dollars of expected losses, expenditures, or payments. The southwestern axis indicates agent 1’s share \( \gamma \) of the loss. Moving along this axis from the western corner to the southern corner increases the extent to which the agent is a victim rather than an injurer. The southeastern axis indicates the relative effectiveness of agent 1’s precaution \( \alpha \). Moving from the southern to the eastern corner increases its effectiveness.

The top surface represents agent \( i \)'s optimized full costs under the mechanism including his expected losses \( L_i(F(m)) \), cost of precaution \( \bar{x}_i \theta_i = m_i F_i(m) \), and liability \( \tau_i(m) \). The final outcome from the mechanism is invariant to agent 1’s initial share of loss \( \gamma \), but varies with the effectiveness of his precaution \( \alpha \). The surface with its southwest edge along the \( \gamma \) axis is agent 1’s optimized cost of care \( m_i F_i(m) \). It is also invariant to the agent’s initial share of loss \( \gamma \), but increases monotonically with the effectiveness \( \alpha \) of his precaution. The third surface from the bottom is optimized social expected loss \( L_s(m) \). The \( \gamma L_s(m) \) surface shows agent 1’s loss increases linearly from zero to \( L_s(m) \) as \( \gamma \) goes from zero to one, for any fixed relative effectiveness of precaution \( \alpha \). The liability \( \tau_1(m) \) surface differs from the top surface by the amount of loss and expenditure on precaution \( \gamma L_s(m) + m_i F_i(m) \) that agent 1 experiences.
directly, so that it offsets both the cost of precaution $m_1 F_1(m)$ and agent 1’s expected loss $\gamma L_S(m)$.

In the western corner of Figure 5, agent 1 bears none of the direct loss $L_S(m)$ and has zero cost of precaution $m_1 F_1(m)$, because his precaution is totally ineffective. His liability equals the full social loss $L_S(m)$ plus agent 2’s costs of precaution $m_2 F_2(m)$. The opposite is true in the eastern corner, where he bears the full loss $L_S(m)$, all the costs of precaution $m_1 F_1(m)$, and has zero liability $\tau_1(m)$. In the southern corner, he bears all the loss $L_S(m)$ but none of the cost of precaution, so his liability equals agent 2’s expenditure on precaution $m_2 F_2(m)$. In the northern corner, he bears no loss but takes all the precaution, and he is liable for the full loss $L_S(m)$. For interior points in the graph, with fixed effectiveness of precaution $\alpha$, agent 1’s liability decreases linearly with his expected loss $\gamma L_S(m)$. For fixed share of loss $\gamma$, his liability decreases with his expenditure on precaution $m_1 F_1(m)$.

Figure 5 has agent 1’s unit cost of precaution normalized to $m_1 = \theta_1 = 1$ and agent 2’s unit cost of precaution set at $m_2 = \theta_2 = 2$. Since the optimized expenditures on precaution $m_1 F_1(m) + m_2 F_2(m)$ equal the social loss from accidents $L_S(m)$, the southwest edge of the $L_S(m)$ surface varies with $m_2$ (with $m_1 = 1$, its vertical intercept is $\sqrt{m_2}$), as does the top surface that is twice the social loss from accidents $2 L_S(m)$. This causes all the other surfaces to shift, while maintaining the same points of intersection with one another relative to the horizontal coordinates. Consequently, Figure 5 represents any value of $m_2 = \theta_2$ relative to $m_1 = \theta_1 = 1$ simply by rescaling the vertical axis so that the southwest edge of the $L_S(m)$ surface has height.
Agents take optimal precaution throughout the diagram, so there is no interpretation as to relative fault. With this parameterization, the liability is always non-negative, so the authority runs a surplus. The authority can decrease the liability by a lump sum amount without destroying the incentives, so it could balance its budget on average over different cases. This seems particularly reasonable when an insurance company operates the mechanism. The liability is designed for efficiency, however, it can compensate victims for their observed accident losses but not for their reported expenditures on precaution.

6.0 Legal policy

These private information liability mechanisms sever the connection between defendants’ payments and the plaintiffs’ compensation. They interpose a third party, the authority, between litigants. This is a significant departure from the justification of property and tort liability as compensatory systems. Does it make sense as a legal policy?

Of course we already have third parties interposed between litigants—they are known as lawyers. Whether the lawyers take a 33% contingency fee, or bill by the hour, they often drive a significant wedge between the rightful victim and her full compensation. The current system leaves determination of “truth” to the judge or jury, without incentivizing the parties to reveal the truth. Legal fees are largely due to the costs of proving the litigants’ private information. Making their payments dependent on their own reports is not incentive compatible with truth revelation. This provides litigants with incentives to exaggerate their injuries or the costs of precaution and mitigation. The current system is not truly compensatory when accounting for the resulting informational rents.

The mechanisms designed here make the payment or compensation independent of the party’s own declaration of private information, giving them no incentive to misrepresent themselves. It retains the justification that payment or subsidy represents the external effect that one party has on the other party as a result of deviating from the rightful activity level. Just as in the current system, there is no liability when the judge or jury sets the initial rights at the efficient level of care and the potential parties satisfy the standard of care. If the authority sets the initial rights at inefficient activity levels, the parties’ reports will nevertheless indicate the truly efficient activity levels, creating an information feedback loop that authority can use to improve future estimates of efficient activity levels (involving future litigants).

\[ \sqrt{m_2}. \]

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24 The average litigation cost of defending a tort claim in which a suit was filed is $35,000 in 2004$ (Hersch and Viscusi, 2007).
In some cases, the law may intentionally choose inefficient standard activity levels based on some existing delineation of rights. In those cases, the parties may adopt the standard activity levels and pay nothing, or choose efficient activity levels and pay (receive) amounts equal to what the other party foregoes (gains) from the deviation from standard activity levels. The profit or utility maximizing activity levels for each party are still the efficient outcomes under the mechanisms.

When the authority bases the payments in these mechanisms on the efficient activity levels, it avoids the discontinuous all-or-nothing jump in liability inherent in pure negligence, while achieving incentives for efficient behavior that comparative negligence lacks. Discrete activity levels may be made arbitrarily small (which requires the exchange of more information) so that deviations within small neighborhoods of the standard activity levels would have smaller payments. Thus, the problem of small changes in behavior that result in large changes in liability is not as problematic under these mechanisms.

The purpose of this paper is not to advocate for the immediate wholesale replacement of nuisance and negligence law, but rather to argue that mechanism design enables a more sophisticated analysis of current law than the Coasean transaction-cost paradigm affords. It often overturns previous results and leads to new insights. Derivation of these results involves a more flexible and realistic economic environment, while using more rigorous economic modeling. Much work remains to develop and test real-world legal mechanisms.

7.0 Conclusion

Coase’s problem of social costs is that social planners often do not know the agents’ private and subjective costs and benefits, which are necessary for determining the Pigouvian taxes and transfers that optimize their activity levels. The Pigouvian paradigm assumes that social planners have omniscient perception of private information and omnipotent enforcement of taxes and transfers. Coase argued that since parties possess private information, they could arrive at jointly optimal outcomes by private bargaining, provided that transaction costs are sufficiently low. He focused on transactions costs as the cause of inefficient outcomes to limit role of law to reducing transaction costs by appropriately delineating the rights to engage in incompatible activities. Relying on private bargaining reduces informational requirements on the social planner, but still requires the law to determine which allocation of rights optimally mitigates the effects of transaction costs. It also assumes that the parties freely and truthfully share their information and ignores their incentives to use information strategically. Rejection of the social planner approach
should not extend to rejecting an authority that designs incentive systems to elicit truthful revelation of the private information that is required to determine efficient outcomes. Coase may be excused for not foreseeing a half-century of advances in the economics of information, game theory, incentive compatibility, and mechanism design—but it is time for law and economics to embrace these developments.

Coase argues for considering all the costs and benefits of alternative institutional arrangements. Mechanism design enables us to explicitly model the sources of these costs and the informational requirements for implementing different alternatives. The Coase paradigm counsels us to consider the transaction costs involved in alternative arrangements and to adopt the one that optimizes the social benefits and costs. This is the correct message of the Coasean paradigm. It provides guidance about how the law ought to be structured. Mechanism design is the best approach to carry the Coasean paradigm forward—based on modern economic theory.

How does the mechanism design approach improve on the Coasean paradigm?

• Mechanism design explicitly models the interaction of the authority with the agents, including informational issues, incentive problems, willingness to participate, budgetary considerations, and strategic behavior. These issues generally do not arise in transaction cost analysis.

• Mechanism design models transactions costs as arising endogenously from the economic environment, so that the mechanism resolves them within the model. The Coasean paradigm usually imposes transaction costs ad hoc.

• Mechanism design derives institutions endogenously with information exchanges, outcome rules, and decentralized decision-making that implements the social objective in explicit equilibria. The Coasean approach chooses an assignment of property rights from a set of exogenously determined alternatives.

• Mechanism design explicitly models the interactions among agents and results in equilibrium outcomes. Coasean outcomes are individually rational and Pareto efficient, but they are not necessarily obtainable as equilibrium outcomes, and they fail to rule out other equilibria.

• Mechanism design provides important implementation possibility and impossibility results that bear on many legal issues. Coasean analysis asserts, without proof, that private bargaining will overcome these problems.

• Mechanism design enables us to formalize Coasean hypotheses to determine the scope of
their applicability.

- Mechanisms permit a wider range of social objectives than profit or utility maximization; including fairness, Rawlsian justice, utilitarianism, or any other well-specified goal. The Coasean approach relies exclusively on efficiency.

Of course not all of the law and economics literature suffers from all of the problems cited here. But these shortcomings are part of what the law and economics field continues to consider acceptable modeling practices. This is particularly problematic when papers assert the impossibility of obtaining truthful revelation of private costs and benefits, ignore the implications of informational rents for designing solutions, and do not model outcomes as economic equilibria.

Models with endogenous transaction costs, information exchanges, and outcome rules that implement social objectives in explicit equilibria provide more rigorous—and realistic—environments for modeling legal institutions. Redesigning the law so that it achieves socially desired outcomes in equilibria with private information and strategic behavior should be the project of meta Coasean law and economics.

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Appendix (For Online Publication)

Proofs of Propositions 1 – 6.

(P1) To prove that this mechanism implements the socially efficient choice in dominant strategies, take the efficiency condition that requires the optimal activities \((x, y)\) be such that:

\[
\theta_x^r + \theta_y^r - h_{xy}^f \geq \theta_m^r + \theta_n^r - h_{mn}^f, \forall m, n.
\]

In particular, this must hold for \(m=x\) and for \(n=y\). So,

\[
\theta_x^r + \theta_y^r - h_{xy}^f \geq \theta_m^r + \theta_n^r - h_{mn}^f, \forall n, \text{and}
\]

\[
\theta_x^r + \theta_y^r - h_{xy}^f \geq \theta_m^r + \theta_n^r - h_{mn}^f, \forall m.
\]

The expression for the liability with the initial rights set to \((u, v)\) is

\[
\tau_{xy}^r = h_{xy}^f - h_{uv}^f.
\]

Substituting \((\tau_{xy}^r + h_{uv}^f)\) for \(h_{xy}^f\), \((\tau_{xn}^r + h_{uv}^f)\) for \(h_{xn}^f\), and \((\tau_{my}^r + h_{uv}^f)\) for \(h_{my}^f\), cancelling terms, and rearranging, we get:

\[
\theta_x^r - (\tau_{xy}^r + h_{uv}^f) \geq \theta_n^r - (\tau_{xn}^r + h_{uv}^f), \forall n, \text{and}
\]

\[
\theta_x^r - (\tau_{xy}^r + h_{uv}^f) \geq \theta_m^r - (\tau_{mn}^r + h_{uv}^f), \forall m.
\]

These are the conditions for \(x\) and \(y\) to be dominant strategies. Thus, social efficiency of \((x, y)\) implies that \((x, y)\) is a (weakly) dominant-strategy equilibrium.

Now start with the last two inequalities and reverse the steps by substituting \(\tau_{mn}^r\) for \((h_{mn}^f + h_{uv}^f)\) and adding \(\theta_x^r\) or \(\theta_y^r\) to both sides of the inequalities to obtain the social efficiency conditions. Thus, that \((x, y)\) is a (weakly) dominant-strategy equilibrium implies that \((x, y)\) is socially efficient. The mechanism implements the socially efficient allocations in weakly dominant strategies.

(P2) To prove that the mechanism implements the socially efficient choice in dominant strategies, start with the efficiency condition, which requires that the optimal activities \((x, y)\) have higher social value than any other activity levels:

\[
\theta_{xy}^i + \theta_{xy}^j \geq \theta_{mn}^r + \theta_{mn}^l, \forall m, n.
\]

In particular, this must hold for \(m=x\) and for \(n=y\). So,

\[
\theta_{xy}^i + \theta_{xy}^j \geq \theta_{mn}^r + \theta_{mn}^l, \forall m, \text{and}
\]

\[
\theta_{xy}^i + \theta_{xy}^j \geq \theta_{xn}^r + \theta_{xn}^l, \forall n.
\]

The expression for the liability with the initial rights set to \((u, v)\) is

\[
\tau_{mn}^i = \theta_{mn}^j - \theta_{uv}^j.
\]
Substituting \((\tau_{mn}^i + \theta_{uv}^i)\) for \(\theta_{mn}^i\), cancelling terms, and rearranging, we get
\[
\theta_{xy}^i - (\tau_{xy}^i + \theta_{uv}^i) \geq \theta_{my}^i - (\tau_{my}^i + \theta_{uv}^i), \forall m, \text{ and} \\
\theta_{xy}^j - (\tau_{xy}^j + \theta_{uv}^j) \geq \theta_{xn}^j - (\tau_{xn}^j + \theta_{uv}^j), \forall n.
\]
These are the conditions for \(i\) and \(j\) to be dominant strategies. Thus, social efficiency of \((x, y)\) implies that \((x, y)\) is a (weakly) dominant-strategy equilibrium.

Now start with the last two inequalities and reverse the steps by substituting \(\tau_{mn}^i\) for \((\tau_{mn}^i + \theta_{uv}^i)\) and adding \(\theta_{x}^i\) or \(\theta_{y}^i\) to both sides of the inequalities to obtain the social efficiency conditions. Thus, \((x, y)\) is a (weakly) dominant-strategy equilibrium implies \((x, y)\) is socially efficient. The mechanism implements the socially efficient allocations in weakly dominant strategies.

\[
(P3) \quad \gamma L_x(\theta_1, \theta_2) + \tau_1(m_1; m_2, \theta) + \theta_1 \hat{x}_1(m_1, m_2) = (1 - \gamma) L_s(\theta_1, \theta_2) + \tau_2(m_2; m_1, \theta) + \theta_2 \hat{x}_2(m_1, m_2).
\]
Start with agent 1’s total cost under the mechanism on the left hand side,
\[
\gamma L_s(\theta_1, \theta_2) + \tau_1(m_1; m_2, \theta) + \theta_1 \hat{x}_1(\theta_1, \theta_2) = (1 + (1 - \alpha)) \left( m_1 \left( \frac{\alpha}{m_1} \right)^{1/2} \left( \frac{\alpha m_2}{(1 - \alpha) m_1} \right)^{1/2} \right) + \\
m_1 \left( \frac{\alpha}{m_1} \right)^{1/2} \left( \frac{\alpha m_2}{(1 - \alpha) m_1} \right)^{1/2} \left( \frac{\alpha m_2}{(1 - \alpha) m_1} \right)^{1/2} \left( \frac{\alpha m_2}{(1 - \alpha) m_1} \right)^{1/2} \\
= \left[ (1 + (1 - \alpha)) \left( \frac{m_1}{\alpha} \right)^{1/2} + m_1 \left( \frac{\alpha}{m_1} \right)^{1/2} \left( \frac{\alpha m_2}{(1 - \alpha) m_1} \right)^{1/2} \left( \frac{\alpha m_2}{(1 - \alpha) m_1} \right)^{1/2} \right] \\
= \left[ (1 + (1 - \alpha)) \left( \frac{m_1}{\alpha} \right)^{1/2} + m_1 \left( \frac{\alpha}{m_1} \right)^{1/2} \left( \frac{\alpha m_2}{(1 - \alpha) m_1} \right)^{1/2} \left( \frac{\alpha m_2}{(1 - \alpha) m_1} \right)^{1/2} \right] \\
= \left[ (1 + (1 - \alpha)) + m_1 \left( \frac{\alpha}{m_1} \right)^{1/2} \left( \frac{\alpha m_2}{(1 - \alpha) m_1} \right)^{1/2} \left( \frac{\alpha m_2}{(1 - \alpha) m_1} \right)^{1/2} \right] \\
= \left[ (1 + (1 - \alpha)) + m_1 \left( \frac{\alpha}{m_1} \right)^{1/2} \left( \frac{\alpha m_2}{(1 - \alpha) m_1} \right)^{1/2} \right] \\
= \left[ (1 + (1 - \alpha)) + m_1 \left( \frac{\alpha}{m_1} \right)^{1/2} \left( \frac{\alpha m_2}{(1 - \alpha) m_1} \right)^{1/2} \right] \\
= (1 + \alpha) \left( \frac{m_2}{(1 - \alpha)} \right)^{1/2} + (1 - \alpha) \left( \frac{m_2}{(1 - \alpha)} \right)^{1/2} \left( \frac{\alpha m_2}{(1 - \alpha) m_1} \right)^{1/2}.
\]
\[
= (1 - \gamma) L_s(\theta_1, \theta_2) + \tau_2(m_2; m_1, \theta) + \theta_2 \hat{x}_2(m_1, m_2),
\]
which is agent 2’s total cost under the mechanism.

(P4) \[\tau_1(m_1; m_2, \theta) = (1 - \gamma) L_s(\theta_1, \theta_2) + \theta_2 \hat{x}_2(m_1, m_2),\]
and

\[
\tau_2(m_2; m_1, \theta) = \gamma L_s(\theta_1, \theta_2) + \theta_1 \hat{x}_1(m_1, m_2).
\]

Start with agent 1’s liability under the mechanism on the left hand side,

\[
\tau_1(m_1; m_2, \theta) = (1 + (1 - \alpha)) \left(\frac{m_1}{\alpha}\right)^{\frac{1}{2}} \left(\frac{m_2}{(1-\alpha) m_1}\right)^{\frac{1-a}{2}} - \gamma \left(\frac{1-a}{m_2}\right)^{-\frac{a}{2}} \left(\frac{\alpha}{m_1}\right)^{-\frac{a}{2}}
\]

\[
= (1 + (1 - \alpha)) \left(\frac{m_1}{\alpha}\right)^{\frac{1}{2}} \left(\frac{m_2}{(1-\alpha) m_1}\right)^{\frac{1-a}{2}} - \gamma \left(\frac{1-a}{m_2}\right)^{-\frac{a}{2}} \left(\frac{\alpha}{m_1}\right)^{-\frac{a}{2}}
\]

\[
= \left((1 - \gamma) \left(\frac{1-a}{m_2}\right)^{-\frac{a}{2}} + (1 - \alpha) \left(\frac{1-a}{m_2}\right)^{-\frac{a}{2}} \left(\frac{\alpha}{m_1}\right)^{-\frac{a}{2}} \left(\frac{1-a}{m_1}\right)^{-\frac{a}{2}}
\]

\[
= (1 - \gamma) L_s(m_1, m_2) + \theta_2 \hat{x}_2(m_1, m_2),
\]

which is agent 2’s loss plus her expenditure on care. The same procedure starting with agent 2’s liability yields agent 1’s loss plus his expenditure on care.

(P5) \[\gamma L_s(\theta_1, \theta_2) + \tau_1(m_1; m_2, \theta) + \theta_1 \hat{x}_1(\theta_1, \theta_2) = 2 L_s(\theta_1, \theta_2),\]
and

\[\gamma L_s(\theta_1, \theta_2) + \tau_2(m_2; m_1, \theta) + \theta_2 \hat{x}_2(\theta_1, \theta_2) = 2 L_s(\theta_1, \theta_2).\]

Start with agent 1’s total cost under the mechanism on the left hand side,

\[
\gamma L_s(\theta_1, \theta_2) + \tau_1(m_1; m_2, \theta) + \theta_1 \hat{x}_1(\theta_1, \theta_2) =
\]

\[
= (1 + (1 - \alpha)) \left(\frac{m_1}{\alpha}\right)^{\frac{1}{2}} \left(\frac{m_2}{(1-\alpha) m_1}\right)^{\frac{1-a}{2}} + m_1 \left(\frac{m_2}{(1-\alpha) m_1}\right)^{\frac{1-a}{2}}
\]

\[
= \left[(1 + (1 - \alpha)) \left(\frac{m_1}{\alpha}\right)^{\frac{1}{2}} + m_1 \left(\frac{m_2}{(1-\alpha) m_1}\right)^{\frac{1-a}{2}}\right]
\]

\[
= \left[(1 + (1 - \alpha)) \left(\frac{m_1}{\alpha}\right)^{\frac{1}{2}} + m_1 \left(\frac{m_2}{(1-\alpha) m_1}\right)^{\frac{1-a}{2}}\right]
\]

\[
= \left[(1 + (1 - \alpha)) \left(\frac{m_1}{\alpha}\right)^{\frac{1}{2}} + m_1 \left(\frac{m_2}{(1-\alpha) m_1}\right)^{\frac{1-a}{2}}\right]
\]

\[
= \left[(1 + (1 - \alpha)) \left(\frac{m_1}{\alpha}\right)^{\frac{1}{2}} + m_1 \left(\frac{m_2}{(1-\alpha) m_1}\right)^{\frac{1-a}{2}}\right]
\]

\[
= 2 \left(\frac{m_2}{(1-\alpha) m_1}\right)^{\frac{1-a}{2}} \left(\frac{\alpha}{m_1}\right)^{-\frac{a}{2}}
\]

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\[= 2 \mathcal{L}_s(\theta_1, \theta_2).\]

By Proposition 3, agent 2’s total cost under the mechanism also equals \(2 \mathcal{L}_s(\theta_1, \theta_2).\)

(P6) \[\theta_1 \bar{x}_1(\theta_1, \theta_2) + \theta_2 \bar{x}_2(\theta_1, \theta_2) = \mathcal{L}_s(\theta_1, \theta_2).\]

By Proposition 5, adding the agents’ total costs under the mechanism equals 4 times the social loss,

\[4 \mathcal{L}_s(\theta_1, \theta_2) = \gamma \mathcal{L}_s(\theta_1, \theta_2) + \theta_1 \bar{x}_1(m_1; m_2, \theta) + \theta_1 \bar{x}_1(m_1, m_2) + (1 - \gamma) \mathcal{L}_s(\theta_1, \theta_2) + \theta_2 \bar{x}_2(m_1, m_2) + \theta_2 \bar{x}_2(m_1, m_2).\]

By Proposition 4, the sum of the liabilities equals the social loss plus twice the expenditure on precaution,

\[\tau_1(m_1; m_2, \theta) + \tau_2(m_2; m_1, \theta) = \mathcal{L}_s(\theta_1, \theta_2) + \theta_1 \bar{x}_1(m_1, m_2) + \theta_2 \bar{x}_2(m_1, m_2).\]

Substituting for the liabilities gives

\[4 \mathcal{L}_s(\theta_1, \theta_2) = 2 \mathcal{L}_s(\theta_1, \theta_2) + 2 \theta_1 \bar{x}_1(m_1, m_2) + 2 \theta_2 \bar{x}_2(m_1, m_2),\] or

\[\mathcal{L}_s(\theta_1, \theta_2) = \theta_1 \bar{x}_1(m_1, m_2) + \theta_2 \bar{x}_2(m_1, m_2).\]