1. Describe the characteristics of suspensions of non-colloidal particles. What kind of motions or interactions should be considered?
2. Describe the motion of an isolated sphere in a simple shear flow.
3. Why does the presence of particles increase the viscosity?
4. Why does viscosity increase linearly with volume fraction in Einstein's viscosity equation?
5. What conditions should be satisfied to apply the Einstein equation? Does it only apply to non-colloidal monodisperse particles?
6. What is the effect on the viscosity of a non-colloidal suspension in the semi-dilute regime of:
    a. particle volume fraction
    b. suspending medium
    c. particle size
    d. particle polydispersity
    e. particle roughness
7. Explain why the sample history is important in determining the viscosity of non-colloidal suspensions?
8. For a non-colloidal suspension, what are the consequences of moving from the dilute to the concentrated regime for the hydrodynamics, the microstructure and the stresses?
9. Is the size of spherical particles important for the viscosity of non-colloidal suspensions?
10. Predict how the components of the flow-induced self-diffusivity $\mathcal{D}$ will change if:
    a. Shear rate doubles
    b. Particle radius doubles
11. What is shear-induced migration, and why do we need to consider it while performing rheological measurements?
12. Describe the features of lubrication hydrodynamics.
13. For non-colloidal particles, what information can be determined from significant values of the normal stress differences N_1 and N_2?
14. In concentrated suspensions, why is the settling rate, V_s, not equal to the settling velocity, $V_{s,0}$ for particles in a dilute suspension?
15. Explain the inertial effects that cause deviation from Stokes flow.
16. Under what conditions is flow reversibility observed in non-colloidal suspensions? Under what conditions is flow irreversibility observed in non-colloidal suspensions?

1. In suspensions of non-colloidal particles it is assumed that there is no Brownian motion. Further, due to the relatively large particle size relative to the range of typical surface interactions, the interparticle forces (electrostatic, steric, etc…) can be ignored except when considering particles near contact. Particle inertia may become important at higher shear rates. Hydrodynamic interactions are generally the most important interaction to consider.
2. An isolated sphere in a sheared fluid will undergo translational motion following the streamline that corresponds to its center in the absence of a particle It will also rotate with a rotational velocity $\Omega = 1/2\dot{\gamma}$.
3. Viscosity corresponds to energy dissipation, which occurs as the results of two mechanisms:
    a) The distortion of the flow field caused by the particle
    b) The friction exerted by the fluid flow at the particle surface
4. Einstein's equation applies only to dilute suspensions for which encounters between particles can be ignored. The viscosity increase is due to the additional energy dissipation from the disturbance of the flow around the particles. As the particles do not interfere with each other, the results of the individual particles are additive. Hence, the viscosity will depend linearly on volume fraction.
5. The Einstein relation applies to dilute suspensions of hard, spherical particles in slow flows (where inertia of fluid and particle can be neglected). The size of the particle is not relevant because there is no other length scale in the problem. As the effect is proportional to the concentration of the particles, the viscosity is proportional to the particle volume fraction. As a consequence, the equation applies to monodisperse and polydisperse suspensions. There is, however, an implicit limiting minimum size: the suspending fluid is to be characterized by a continuum property (viscosity), which requires that the particle should be sufficiently large as compared to the fluid molecules such that, on the size of a particle, the fluid can be considered a continuum (see also Chapter 10). The Einstein equation applies equally well to colloidal hard spheres as there are no additional restrictions concerning Brownian motion.
6. The Einstein relation is only valid as long as interactions between particles can be neglected. However, in the semi-dilute regime, the average distance between particles becomes comparable to the particle size. Therefore, the flow field around particles will be significantly affected by the neighboring particles and this acts to increase the rate of energy dissipation.
    a. The effect of hydrodynamic interactions depends on the interaction between pairs of particles and, hence, will be proportional to the square of the volume fraction.
    b. The suspension viscosity remains proportional to the viscosity of the suspending medium.
    c. In the absence of any significant interparticle forces other than hydrodynamic interactions, particle size is not relevant to the viscosity of non-colloidal dispersions.
    d. Polydispersity effects become evident in semi-dilute suspensions. The order Φ^2 term depends on the size distribution, unlike the intrinsic viscosity. This can be

understood because the hydrodynamic forces depend on the distances between particles relative to the particle size. Calculation of bimodal and polydisperse non-Brownian suspensions show that the viscosity decreases with increasing polydispersity in the semi-dilute regime.

e. Surface roughness can lower the viscosity of a non-colloidal suspension by reducing or elimination the ability of particles to come into close approach where the lubrication stresses are important. Roughness will also break the fore-aft symmetry of the trajectories, causing anisotropy in the microstructure and introducing additional rheological phenomena such as viscosity variation upon flow reversal or hysteresis.

7. The stresses depend on the instantaneous microstructure. As the hydrodynamics are deterministic, in the absence of Brownian motion the microstructure would theoretically always be determined by the initial microstructure and the sample's subsequent flow history. This effect complicates the calculation of the viscosity by the method of trajectories, for example, because the result depends on the initial probability of finding particles in the region of closed trajectories.
8. As suspensions become more concentrated many-body hydrodynamic interactions become important. Such many body interactions greatly increase the viscosity and also lead to shear-induced diffusion. Shear induces an anisotropy in the microstructure and this leads to finite normal stress differences. N_1 and N_2 are both negative (in contrast to the case of polymers) and the magnitude of N_1 is less than that of N_2.
9. For suspensions of non-colloidal spherical particles the viscosity is independent of the absolute particle size, but it will change with the size distribution. For dilute suspensions, particle size distribution is not relevant. For concentrated suspensions, polydispersity in particle size reduces the suspension viscosity significantly as broadening the size distribution increases the maximum packing. The absolute maximum packing fraction, ϕ_{max}, for monodisperse hard spheres is approximately .74 (face-centered cubic lattice, FCC). The suspension, however, would be unable to flow at this packing fraction. The largest ϕ_{max} at which a suspension of monodisperse hard spheres will still flow is .64, corresponding to random maximum packing. Polydisperse systems can have higher ϕ_{max}. This is because spheres of varying size can pack more densely. The smaller particles can fit in open spaces between the larger particles. The consequence of a higher ϕ_{max} include a lower viscosity at the same ϕ. The drop in viscosity relative to monodisperse suspensions becomes more significant with increasing volume fraction and size ratio.
10. The components of the flow-induced self-diffusivity scale are determined by the ratio of the square of a characteristic length over time. Here, they scale with $\dot{\gamma}a^2$, as $\dot{\gamma}^{-1}$ and a are the only time and length scales in the problem. Therefore, if shear rate doubles, the components of $\mathcal{D}_s$ will double. If particle size doubles, the components of $\mathcal{D}_s$ will quadruple.
11. When there are spatial variations in the gradients in shear rate or shear stress, the particles migrate from regions of high shear to low shear. This is termed shear-induced migration and results in a concentration gradient required to balance the shear-induced diffusion. In this manner a steady state concentration gradient, and consequently also a viscosity gradient is generated. The viscosity then varies with position, which would affect the velocity profiles in the suspension.

12. The hydrodynamic force exerted on the spheres to squeeze them together at a relative speed U is given by eqn. (2.14). This equation is valid in the lubrication limit, where $h << (a_1 + a_2)$. It can be seen that the force required to bring two particles together at constant viscosity changes as $1/h$ and therefore diverges to infinity when the particles approach one another. The stress in the fluid between the particles is proportional to the forces acting on the particles. Hence, the hydrodynamic lubrication stresses can be significant when the average particle separation becomes small.
13. In non-Brownian suspensions of hard spheres, the suspending medium is assumed to be Newtonian and thus would be devoid of normal stress differences. However, to have values of N_1 and N_2 that can be measured and that are significant, the suspension should have an anisotropic microstructure under flow. Conversely, an isotropic microstructure would yield zero normal stress differences.
14. Hydrodynamic interactions between particles will act to hinder settling. An important effect is that for sedimentation to occur, a backflow of the liquid phase is required. For a single particle, or for a dilute suspension of particles in a dilute suspension, it can be assumed that the particles do not hinder the back flow arising from each other. However, in semi-dilute and concentrated solutions, particles will interact with each other, and hinder the back flow.
15. When the fluid velocity increases such that the particle Reynolds number becomes significant, then inertial effects can influence the fluid flow around particles. Additional migration effects such as particle lift can occur. One such effect, called tubular pinch, occurs in pressure-driven tube flow. Segre and Silberberg found that in a tube of radius R, particles congregated in a ring of radius 0.6R. The resulting concentration profile is a function of concentration, Reynolds number, and relative particle size. Particle inertia can also be important. The ratio of particle inertia forces to viscous forces is known as the Stokes number, and can help to estimate when particle inertia effects become significant in the suspension. When large, heavy particles are in low viscosity media, St can become high and particle collisions dominate normal viscous dissipation. Additionally, strong effects of particle inertia have been observed on flows with non-zero Stokes numbers and Reynolds numbers of zero.
16. Flow reversibility is observed in very dilute, non-colloidal suspensions (ϕ=0.05) with smooth particles at low Re. The particles are assumed not to interact and hence the system is close to perfectly deterministic. Flow irreversibility will be induced by effects such as particle surface roughness, many-body particle interactions, and inertia. In more concentrated systems many-body hydrodynamic interactions break the symmetry of the particle trajectories and lead to irreversible, time dependent behavior (such as shear-induced particle diffusion).