Applications of Mathematics To Economics

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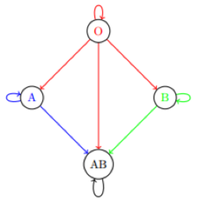
Abstract

The top trading cycle (TTC) algorithm appeared in the 1973 paper of Lloyd Shapley and Herbert Scarf where it was used to solve a problem of how to allocate indivisible goods, such as houses, to different people. In recent years, the TTC algorithm has been used to solve problems such as school choice, matching medical residents to hospitals, and matching patient to kidney donors. This summer, we have analyzed the work of Alvin Roth, Tayfun Sönmez, and M. Utku Ünver in applying this algorithm to the kidney exchange markets. In the kidney exchange market it is never known when a cadaveric kidney will be available, which adds uncertainty to the problem. Therefore, kidneys cannot be allocated in the exact same way that houses are. Because of this Roth, Sönmez, and Ünver developed the top trading cycle and chains (TTCC) mechanism which efficiently matches kidney patients with donors using cycles and w-chains. This mechanism could increase the number of live donor and patient matchings, which will decrease the waitlist for cadaver kidneys.

Background

- Donations from live donors have a higher chance of success
- There are different ways to receive a kidney donation
  - From a live donor
  - Paired exchange
  - Indirect exchange
- The four major blood types are O, A, B, and AB
- Cadaveric kidneys are assigned based on a point system



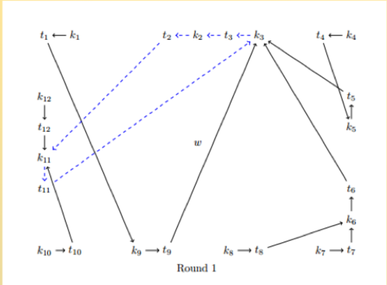
Top Trading Cycle (TTC) Algorithm

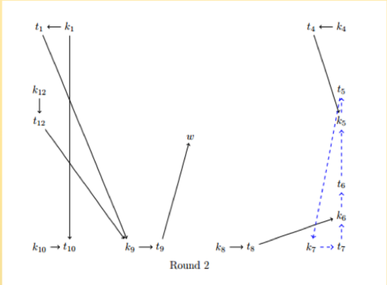
The top trading cycle (TTC) algorithm finds a way to allocate indivisible goods among people without the use of money.

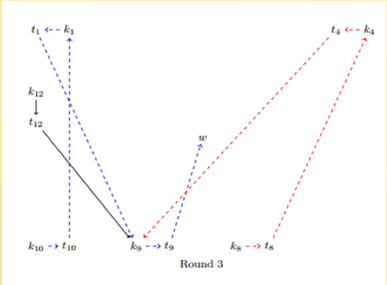
People's ranking for houses from best (left) to worst (right)

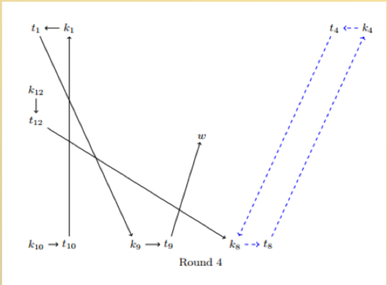
1: (h_3, h_2, h_4, h_1)
2: (h_4, h_1, h_2, h_3)
3: (h_1, h_4, h_3, h_2)
4: (h_3, h_2, h_1, h_4)

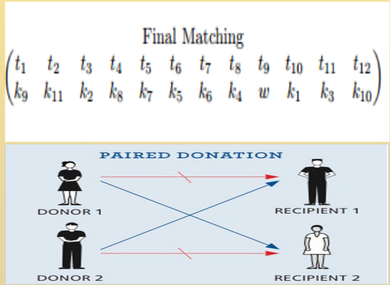
Kidney Exchange Graphs

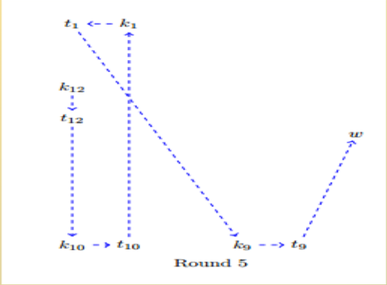












Top Trading Cycles and Chains (TTCC)

- No kidney can be assigned to more than one person
- Many patients can be assigned to the cadaveric waitlist

1. Active patients point to their most preferred kidney, and kidneys point to their paired recipient
2. Remove any cycles and have remaining patients point to their most preferred kidney. Repeat until no cycles remain
3. If no pairs remain we are finished, otherwise select one w-chain using the chain selection rule
4. Repeat steps 2 and 3 until no pairs remain

Kidney Exchange Problem

Here are the preferences of the 12 patients:

- $t_1 : k_9, k_{10}, k_1$
- $t_2 : k_{11}, k_3, k_5, k_6, k_2$
- $t_3 : k_2, k_4, k_5, k_6, k_7, k_8, w$
- $t_4 : k_5, k_9, k_1, k_8, k_{10}, k_3, w$
- $t_5 : k_3, k_7, k_{11}, k_4, k_5$
- $t_6 : k_3, k_5, k_8, k_6$
- $t_7 : k_6, k_1, k_3, k_9.k_{10}, k_1, w$
- $t_8 : k_6, k_4, k_11, k_2, k_3, k_8$
- $t_9 : k_3, k_{11}, w$
- $t_{10} : k_{11}, k_1, k_4, k_5, k_6, k_7, w$
- $t_{11} : k_3, k_6, k_5, k_{11}$
- $t_{12} : k_{11}, k_3, k_9, k_8, k_{10}, k_{12}$

Our chain selection rule is choose the longest w-chain. If there is a tie choose the one with the highest priority patient. If there is another tie choose the one with the second highest priority patient, and so on. Keep the selected w-chain until termination.

Efficiency

- Keep the selected w-chain until the end
- Choose minimal w-chains and remove them
- Choose w-chain with highest priority patient
- Prioritize patient-donor pairs so that pairs with type O patients have higher priority

Future Work

To make this work really beneficial there needs to be more kidneys available on the market. This can be accomplished by automatically making everyone a part of the donor registry. Another possibility is providing financial compensation such as a discount on your driver license if you join the registry, or paying for the deceased's funeral if their kidneys are donated. We could also follow Iran's lead and make the buying and selling of kidneys legal.

References

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3. Roth, Alvin E., Tayfun Sönmez, and M. Utku Ünver. "Kidney Exchange." *The Quarterly Journal of Economics* Vol. 119, No. 2 (2004): 457-488. Web. 18 Jul. 2016.