# Network Addressing Using Biclique Graph Decomposition

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## Introduction

Graphs are abstract models of networks and are used in many real-world applications such as web-page ranking, network design and routing and error-correcting codes. A graph G consists of a set of vertices V and a set of edges E which are pairs of vertices. In the example below, the vertices are the airports and the edges are the pairs of airports connected by a direct flight.

Figure 1.

## Addressing of a Graph

In 1971 while at Bell Labs, Graham and Pollak introduced the problem of labeling or addressing nodes in a network such that a message can be transmitted via a shortest route by looking at the labels of the neighbors of the current node. Graham and Pollak suggested giving each vertex an address which is a {0,1,*} word of length N such that the distance in the graph between any two vertices u and v equals the number of positions in their addresses where one vertex has a 0 and the other has a 1. Graham and Pollak studied the minimum or optimal number N of the length of an addressing of various graphs and they determined this parameter for complete graphs, trees and cycles. There are very few other graphs for which this parameter has been determined exactly. In our research, we determine the minimum length of an addressing of the Lattice graphs, Hamming graphs, the complete graph K2,3, and we obtain the best known addressing of the triangular graph T(5).

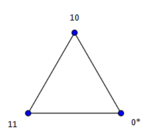


Figure 2.

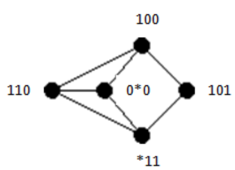


Figure 3.

## Finding Optimal Addressings

In 1971, Graham and Pollak proved that N equals the minimum number of bicliques (complete bipartite subgraphs) that partition the edge multiset of the distance multigraph of G. The distance multigraph of G has the same vertex set as G and the multiplicity of an edge uv equals the distance between u and v in G. Using this fact, Graham and Pollak showed that N>=h(D), where D is the distance matrix and h(D) is the maximum of the numbers of positive and negative eigenvalues of D. When N=h(D), then h(D) is the optimal length of an addressing.

## Lattice Graphs

Here is an example of the 3-by-3 lattice graph in which every 2 vertices in the same row or column are connected by an edge.

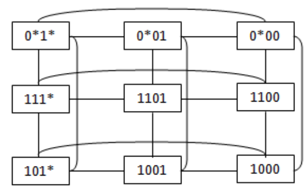


Figure 4.

This same pattern for the bicliques can be used for the 4-by-4 lattice graph:

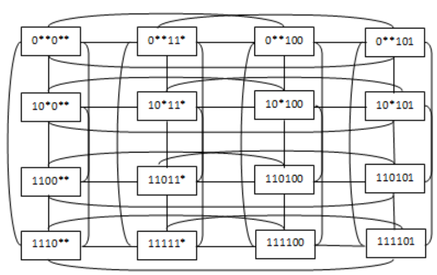


Figure 5.

## Addressing K2,3

This bipartite graph has 5 vertices 3 on one side, two on the other. The eigenvalue bound found by looking at the adjacency matrix is 3. However, we proved algebraically that the best possible addressing has a length of 4, shown below.

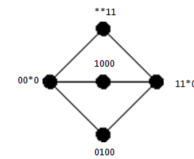


Figure 7.

## Triangular Graph T(5)

We first labeled the vertices as 2-subsets of {1, 2, 3, 4, 5}, then used biclique graph decomposition. The best addressing we could find was using 6 bicliques. Below shows how the vertices were labeled and the biclique decomposition:

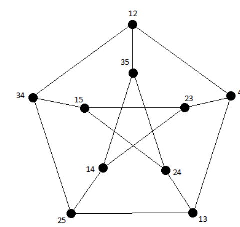


Figure 6.

1) 12, 13, 14, 15/ 34, 24, 23, 25, 35, 45
2) 12, 25/ 34, 13, 14, 35, 45
3) 23, 24/ 15, 25, 34, 35, 45
4) 13, 23, 35/ 14, 24, 45
5) 15/ 12, 13, 14, 34
6) 34/ 25, 35, 45

## Work Cited

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