MATH829
Linear and Semidefinite Programming in Combinatorial Optimization
Spring 2016

Lectures Mondays, Wednesdays, Fridays 10.10-11am Memorial Hall 126
Instructor Dr. Sebastian Cioabă, `cioaba@udel.edu` Ewing Hall 506
Office Hours Tuesdays and Thursdays 9.30-10.30am or by appointment.

Course Description and Objectives

Linear programming is one of the most fundamental tool in optimization and discrete mathematics with numerous applications. Linear programs are efficiently solvable and have a powerful duality theory. An important method in combinatorial optimization is to write the problem as a linear program with integer variables. We then forget the integrality constraints and we obtain a linear relaxation which can be solved efficiently. Linear programs are special cases of convex programs. Semidefinite programs are more general convex programs to which many of the nice properties of linear programs extend. These semidefinite relaxations are still efficiently computable, but approximate the optimum much better. Semidefinite programming methods have been successful in obtaining the best possible results for several important problems and some of the highlights include Lovász work in computing the Shannon capacity of a graph and Goemans-Williamson 0.878 approximation for the max-cut of graph (the maximum number of edges between the two parts of a partition of the vertex set). The course will be introduction to the linear and semidefinite programming methods and their applications in combinatorial optimization.

The course will introduce the students to the linear programming method by the basics of this method and its use in combinatorial optimization via fractional variants of classical parameters such as vertex-cover, independence number, chromatic number, matching number for graph theory or Delsarte's LP bound for association schemes and coding theory. The course will introduce the students to the basics of semidefinite programming method and its applications in combinatorial optimization via Lovász' work on Shannon capacity, Goemans-Williamson on max-cut and time permitting, Schrijver's work in coding theory. Applications of semidefinite programming in to other areas such as association schemes, extremal combinatorics and topological graph theory may be discussed.

Topics

Linear programming basics (linear program, simplex method, Farkas lemma, duality, applications to fractional graph theory, max-weight matching, polyhedral combinatorics, fractional chromatic number, Delsarte's linear programming bound in association schemes and coding theory).

Semidefinite programming basics (semidefinite program, semidefinite version of Farkas lemma, semidefinite duality, algorithms for semidefinite programming (ellipsoid method and interior point/barrier method), Shannon capacity, Lovász theta function, Goemans-Williamson approximation of maxcut)

Textbooks and Resources

The books of Bernd Gärtner and Jiri Matoušek books below.

References

[1] Jiří Matoušek and Bernd Gärtner, Understanding and Using Linear Programming, Springer 2007.

[2] Bernd Gärtner and Jiří Matoušek, Approximation Algorithms and Semidefinite Programming, Springer 2012.

Other resources are:

- Vašek Chvátal *Linear programming* A Series of Books in the Mathematical Sciences. W. H. Freeman and Company, New York, 1983. xiii+478 pp. ISBN: 0-7167-1195-8; 0-7167-1587-2

- A. Schrijver *Theory of linear and integer programming.* Wiley-Interscience Series in Discrete Mathematics. A Wiley-Interscience Publication. John Wiley & Sons, Ltd., Chichester, 1986. xii+471 pp. ISBN: 0-471-90854-1

- L. Lovász *Semidefinite programs and combinatorial optimization* Recent advances in algorithms and combinatorics, 137–194, CMS Books Math./Ouvrages Math. SMC, 11, Springer, New York, 2003.

- The lecture notes of Monique Laurent and Frank Vallentin:

  `https://sites.google.com/site/mastermathsdp/lectures`

- The lecture notes of Ryan O'Donnell:

  `http://www.cs.cmu.edu/afs/cs.cmu.edu/academic/class/15859-f11/www/`

- The lecture notes of Bill Martin:

  `http://users.wpi.edu/ martin/RESEARCH/CIMPA/`

Grading Scheme

Your final grade will be calculated based on your performance in:

Homework I will assign homeworks throughout the semester. The homework will contribute 40% to your final grade.

Exams I plan to have 2 exams throughout the semester: Monday, March 21st and Friday, May 13. The exams will be held during class. Each exam will be worth 15% of your final grade.

Research Paper Each of you will pick a research paper and will write a ≤ 4 pages summary of it. I will meet with each of you individually for 10 minutes and ask you questions about the paper. Based on your write-up and your *interview*, you will receive a grade which can contribute up to 20% to your final grade.

Computing Project I plan to assign a computing project. You may use the cvx package in Matlab or cvxopt in Sage (see `http://web.cvxr.com/cvx/doc/sdp.html` or `http://cvxr.com/cvx/` or `http://cvxopt.org/`) or other packages to setup linear and semidefinite programming relaxations of known combinatorial parameters and obtain numerical estimates for such parameters. This will be worth 10% of your grade.

We will have no final exam. The correspondence between the number grade and the letter grade is the following:

A(90-100), A-(85-90), B+(80-85), B(75-80), B-(70-75), C+(65-70),C(60-65),C-(55-60), D(50-55),F(less than 50).

<u>Faculty Statement on Disclosures of Instances of Sexual Misconduct</u>

If, at any time during this course, I happen to be made aware that a student may have been the victim of sexual misconduct (including sexual harassment, sexual violence, domestic/dating violence, or stalking), I am obligated by federal law to inform the university's Title IX Coordinator. The university needs to know information about such incidents to, not only offer resources, but to ensure a safe campus environment. The Title IX Coordinator will decide if the incident should be examined further. If such a situation is disclosed to me in class, in a paper assignment, or in office hours, I promise to protect your privacy--I will not disclose the incident to anyone but the Title IX Coordinator.

For more information on Sexual Misconduct policies, where to get help, and reporting information please refer to www.udel.edu/sexualmisconduct.

At UD, we provide 24 hour crisis assistance and victim advocacy and counseling. Contact 302-831-2226, Student Health Services, to get in touch with a sexual offense support advocate.