total 90 min

Exam 2, Phys208 December 12, 2011

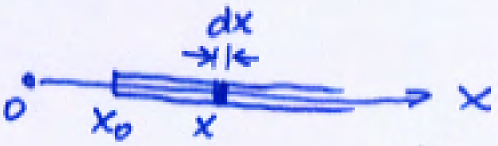
Name: ______________________ **Section #** ____________

1. **(10 pts)** A line of charge starts at $x = +x_0$ and extends to positive infinity. The linear charge density is $\lambda = \lambda_0 x_0 / x$, where λ_0 is a constant.

5 min

(a) (5pts) Determine the electric field at the origin. (Use any variable or symbol stated above along with the following as necessary: k_e.)

(b) (5 pts) Determine the electrical potential at the origin.



(a) $$E = E_x = \int_{x_0}^{\infty} \frac{k_e\,dq}{x^2}$$

$$E_x = k_e \int_{x_0}^{\infty} \frac{\lambda\,dx}{x^2}$$

$$= k_e \int_{x_0}^{\infty} \frac{\frac{\lambda_0 x_0}{x}\,dx}{x^2}$$

$$= k_e \lambda_0 x_0 \int_{x_0}^{\infty} \frac{dx}{x^3}$$

$$= -\overset{\frac{1}{2}}{\cancel{2}} k_e \lambda_0 x_0 \left(\frac{1}{\infty^2} - \frac{1}{x_0^2}\right)$$

$$= \frac{\cancel{2} k_e \lambda_0}{2 x_0}$$

$\lambda_0 > 0$ E in negative x-dir

$\lambda_0 < 0$ E in positive x

(b) $$V = \int_{x_0}^{\infty} \frac{k\,dq}{x}$$

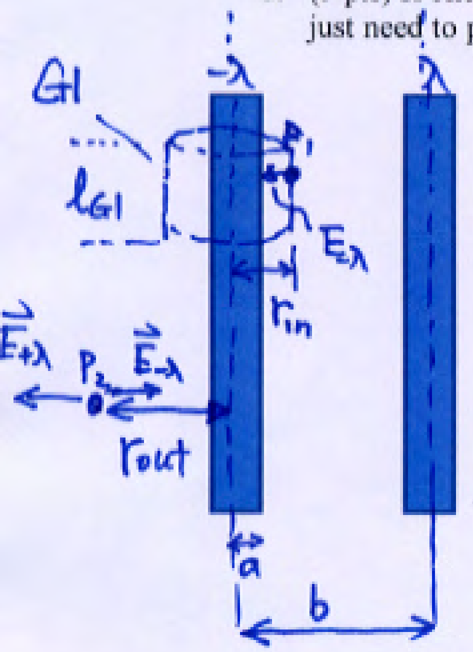
$$= k_e \lambda_0 x_0 \int_{x_0}^{\infty} \frac{dx}{x^2}$$

$$= -k_e \lambda_0 x_0 \left(\frac{1}{\infty} - \frac{1}{x}\right) = k_e \lambda_0$$

20 min

2. (15pts) Two parallel long wires of radius a carry opposite charges with linear charge density of $-\lambda$ (left wire) and $+\lambda$ (right wire), as shown in the figure below. The center-to-center distance of two wires is b.

   a. (6pts) Use Gauss' law to calculate the electric field between and outside the two wires (you do not need to calculate the field inside the wire. You need show your work, just write down the answer is not going to earn any credits).
   b. (4 pts) Evaluate the potential difference between two wires.
   c. (2 pts) Calculate the capacitance per unit length.
   d. (3 pts) If one of the wires is grounded, will the answers to (a) and (b) change (you just need to provide your reasoning, no calculation is needed)



(a) Let's calculate E fields at P_1 & P_2, P_3

Using the Gaussian Surface G_1

$$\oint \vec{E}_{-\lambda} \cdot d\vec{A} = \frac{\delta_{enc,G1}}{\varepsilon_0}$$

$$E_{-\lambda} \cdot 2\pi r_{in} \cdot l_{G1} = \frac{|\lambda| \cdot l_{G1}}{\varepsilon_0}$$

$$E_{-\lambda} = \frac{\lambda}{2\pi \varepsilon_0 r_{in}}$$ point towards Left

Similarily: $$E_\lambda = \frac{\lambda}{2\pi \varepsilon_0 (b - r_{in})}$$

also point towards left

the total field ~~inside~~ in between two wires

$$E_{in} = E_{-\lambda} + E_\lambda = \frac{\lambda}{2\pi\varepsilon_0}\left[\frac{1}{r_{in}} + \frac{1}{b - r_{in}}\right]$$

towards left

at P_2 $$E_{-\lambda} = \frac{\lambda}{2\pi \varepsilon_0 r_{out}}$$ point to right

$$E_{+\lambda} = \frac{\lambda}{2\pi\varepsilon_0 (r_{out} + b)}$$ point to left

total

$$E_{out} = E_{-\lambda} - E_{+\lambda} = \frac{\lambda}{2\pi\varepsilon_0}\left[\frac{1}{r_{out}} - \frac{1}{b + r_{out}}\right]$$

next

Cont. E_{out} point towards right

Similarly E_{out} at P_3

$$E_{out} = \frac{\lambda}{2\pi\varepsilon_0}\left[\frac{1}{r_{out}} \div \frac{1}{r_{out}+b}\right]$$ also towards right

where r_{out} is the distance to $+\lambda$ wire.

(b) $$V_{\lambda,-\lambda} = \int_a^{\cancel{b-2a}\ b-a} \vec{E}_{in}\cdot d\vec{r}$$

$$= \int_a^{b-2a} \frac{\lambda}{2\pi\varepsilon_0 (b-r_{in})}\cdot dr_{in}$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{b-\cancel{2}a}{a}$$ (we did not worry about the sign)

the potential is always high at $+\lambda$ wire.

(c) $$\frac{C}{\ell} = \frac{Q/V_{\lambda,-\lambda}}{L} = \frac{\lambda}{V_{\lambda,-\lambda}} = \frac{2\pi\varepsilon_0}{\ln\frac{b-\cancel{2}a}{a}}$$

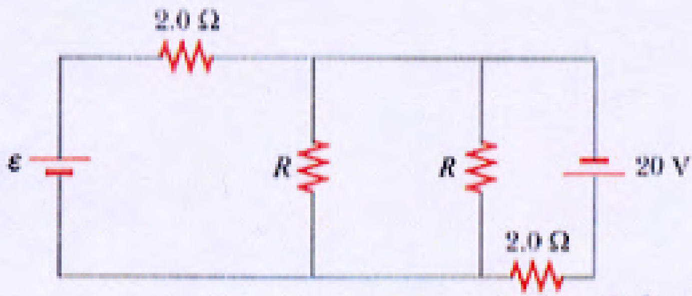
(d) If one of wires is grounded, the E field between the wires will be altered since it loses the contribution from one wire. Therefore both E field (a) and potential difference (b) will change.

※

12 min

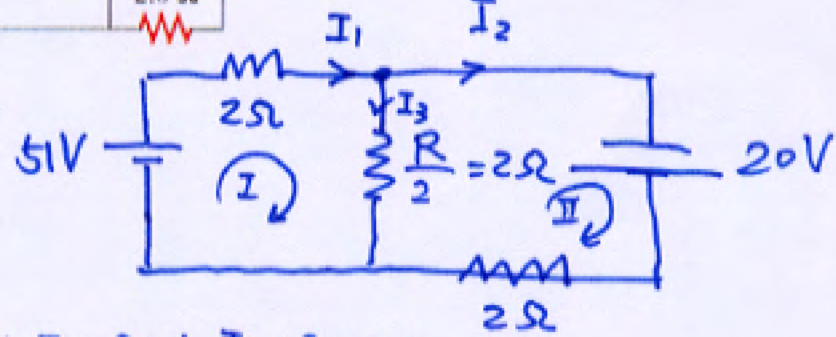
3.(10pts) (a) (6pts) Calculate the power delivered to resistor $R = 4.0\ \Omega$ (any one of two) in the figure ($\varepsilon = 51$ V) (you must show your work).

(b) (4pts) calculate the power delivered by each battery.



Equivalent Circuit

two R's in parallel



KVL Loop I $\quad -51 + I_1 \times 2.0 + I_3 \cdot 2.0 = 0$

Loo II $\quad -20 + I_2 \cdot 2.0 - I_3 \times 2.0 = 0$

KCL $\quad I_1 = I_2 + I_3$

re-arrange the Eqs

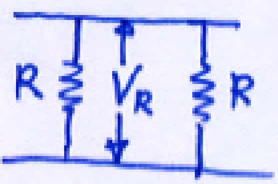
(1) $\quad 2.0 I_1 + 0 I_2 + 2.0 I_3 = 51$

(2) $\quad 0 I_1 + 2.0 I_2 - 2.0 I_3 = 20 \Rightarrow$

(2) $\quad I_1 - I_2 - I_3 = 0$

$$I_3 = \frac{\begin{vmatrix} 2.0 & 0 & 51 \\ 0 & 2.0 & 20 \\ 1 & -1 & 0 \end{vmatrix}}{\begin{vmatrix} 2.0 & 0 & 2.0 \\ 0 & 2.0 & -2.0 \\ 1 & -1 & -1 \end{vmatrix}}$$

$$= \frac{-51 \times 2.0 + 2.0 \times 2.0}{-4.0 - 4.0 - 4.0} = 5.17\ (A)$$



$$V_R = I_3 \cdot \frac{R}{2} = 10.34\ (V)$$

$$P_R = \frac{V_R^2}{R} = \frac{10.34^2}{4} = 26.7\ W.$$

(b) From Eq (1) $\quad I_1 = \frac{51 - 2I_3}{2} = 20.33\ (A) \Rightarrow P_{51V} = I_1 \mathcal{E} = 1036\ (W)$

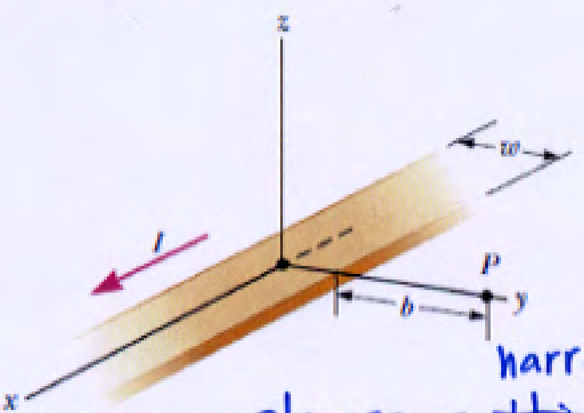
From Eq(3) $\quad I_2 = I_1 - I_3 = 15.16\ (A)$

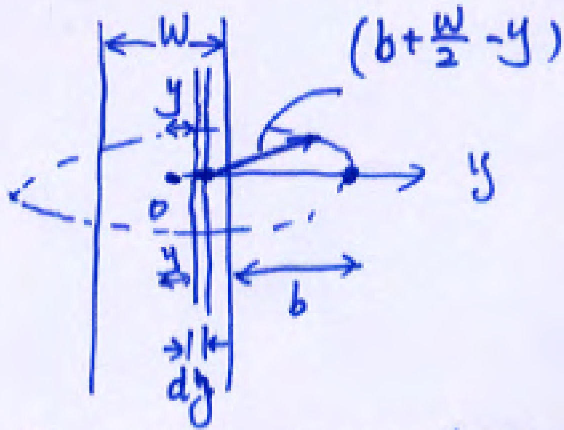
$$P_{20} = I_3 \mathcal{E} = 303\ (W)$$

#

15 min

4 (10 pts) A very long, thin strip of metal of width w carries a current I along its length as shown in the figure below. The current is distributed uniformly across the width of the strip. Find the magnetic field at point P in the diagram. Point P is in the plane of the strip at distance b away from its edge. (Use any variable or symbol stated above along with the following as necessary: μ_0 and π.)





(1) Choose a ~~thin~~ narrow strip y position away from the ~~w~~ center, the ~~str~~ narrow strip has the widty of dy as shown in above figure.

(2) build an Ampere's loop with the narrow strip at the center. the radius is $r = b + \frac{w}{2} - y$

(3) Apply Ampere's law denoting the magnetic field from the narrow strip is ΔB

$$\oint \vec{\Delta B}\cdot d\vec{s} = \mu_0 i_{enc} \qquad \Delta B \cdot 2\pi r = \mu_0 I \cdot \frac{dy}{w}$$

$$\Delta B = \frac{\mu_0 I}{2\pi w}\frac{dy}{r} = \frac{\mu_0 I}{2\pi w}\frac{dy}{b+\frac{w}{2}-y}$$

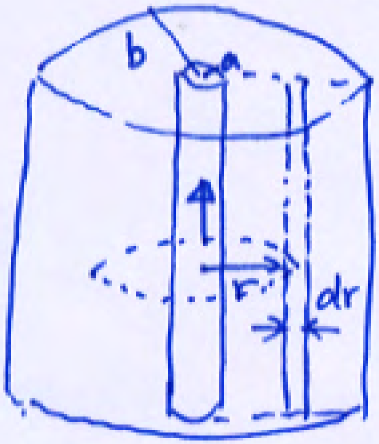
the total magnetice field from the thin strip

$$B = \int \Delta B = \frac{\mu_0 I}{2\pi w}\int_{-\frac{w}{2}}^{\frac{w}{2}} \frac{dy}{b+\frac{w}{2}-y} = -\frac{\mu_0 I}{2\pi w}\ln\left(b+\frac{w}{2}-y\right)$$

$$= \frac{\mu_0 I}{2\pi w}\ln\frac{b+w}{b} = \frac{\mu_0 I}{2\pi w}\ln\left(1+\frac{w}{b}\right)$$

10 mins

5 **(10pt)** Calculate the inductance per unit length of a coaxial cable with inner and outer diameters of a and b, respectively.



(1) assume a current I is flowing along the center conductor

(2) Calculate the flux throug a narrow strip of r distance away from the center & dr width

First calculate $B(r)$ at this narrow strip

use Ampere's Law $\oint \vec{B}(r) \cdot d\vec{s} = \mu_0 I_{enc}$

$$B(r) \cdot 2\pi r = \mu_0 I$$

$$B(r) = \frac{\mu_0}{2\pi} \frac{I}{r}.$$

$d\Phi_B = B(r) \cdot \ell \, dr$ where ℓ is the length of the cable

$$\Phi_B = \int_a^b d\Phi_B = \frac{\mu_0 I_0 L}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 I_0 \ell}{2\pi} \ln \frac{b}{a}$$

(3) Inductance per unit length

$$\frac{L}{\ell} = \frac{\Phi_B / \ell}{I} = \frac{\mu_0}{2\pi} \ln \frac{b}{a}$$

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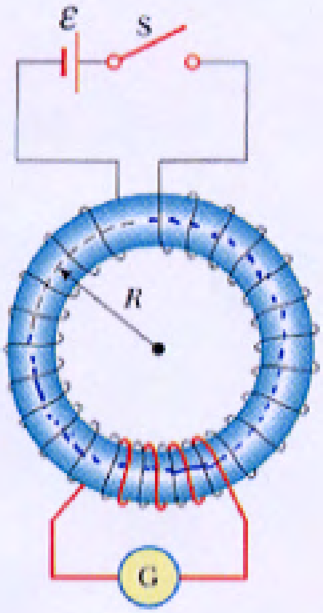
3 min   The cross section area is 3.14×10

6. (20pts) A toroid has a mean radius of 20.0 cm and 600 turns. A secondary coil of 50 turns is connected to a voltmeter (G)
(a) (5pt) If the current in the windings is 3.00 A, use Ampere law to find B (assumed uniform) inside the toroid. (you must show your work for credit).
(b) (5pts) calculate the inductance of the toroid.
(c) (2pts) Calculate the voltage on the secondary coil.
(d) (8pts) If the battery is replaced with i(t)=2sin(624t+30°)A, calculate the voltage on the secondary coil.



(a) $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$

$B\cdot 2\pi r = \mu_0 NI$

$B = \dfrac{\mu_0 NI}{2\pi r} = \dfrac{4\pi \times 10^{-7} \times 600 \times 3}{2\pi \cdot 0.2}$

$= 1.8\times 10^{-3}\ (T)$

(b) $\Phi_B = NBA = 600 \times 1.8\times 10^{-3} \times 3.14\times 10^{-4}$

$= \cancel{1.884}\ 3.39 \times 10^{-4}\ (wb)$

$L = \dfrac{\Phi_B}{I} = \cancel{6.28}\ 1.13 \times 10^{-5}\ (H) = \cancel{62.8}\ 11.3\ \mu H$

(c) ~~Since~~ $\cancel{d\Phi_B}$ Since $\dfrac{d\Phi_B}{dt} = 0 \Rightarrow V = 0.$

(d) $\Phi_B = N_S AB = N_S A \cdot \dfrac{\mu_0 N i}{2\pi r}$

$v = -\dfrac{d\Phi_B}{dt} = -\dfrac{N_S N A \mu_0}{2\pi r}\dfrac{di}{dt} = -\dfrac{N_S N A \mu_0}{2\pi r}\cdot 2\cos(624t+30°)$ $\times 624$

$= \dfrac{50 \times 600 \times 3.14\times 10^{-4} \times 4\pi \times 10^{-7}}{2\pi \cdot 0.2}\cdot 2\cos(624t+30°) \times$

$= \cancel{5.65\times 10^{-5}\cos(624t+30°)}$

$= \cancel{0.035}\ 0.012 \cos(624t+30°)\ (V)$

#

5 min

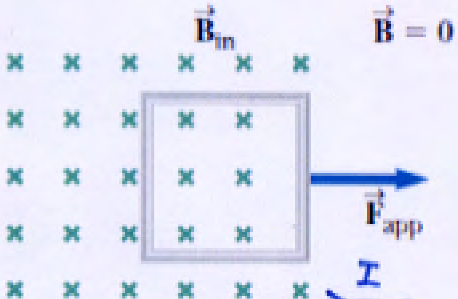
7. (15pt) (31-P64) An N-turn square coil with side ℓ and resistance R is pulled to the right at constant speed v in the presence of a uniform magnetic field B acting perpendicular to the coil as shown in the figure below. At $t = 0$, the right side of the coil has just departed the right edge of the field. At time t, the left side of the coil enters the region where $B = 0$. In terms of the quantities N, B, ℓ, v, and R, find symbolic expressions for the following.

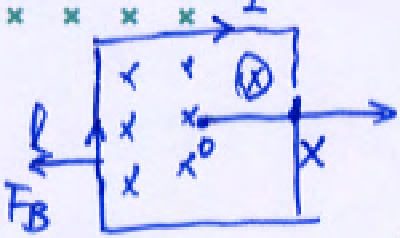
(a) (5pt) the magnitude of the induced emf in the loop during the time interval from $t = 0$ to t

(b) (3pt) the magnitude of the induced current in the coil and the direction

(c) (2pts) power delivered to the coil

(d) (5pts) the force required to remove the coil from the field and the direction





(a) $\Phi_B = BA$

$= B \cdot \ell(\ell - x)$

$\text{emf} = -\dfrac{d\Phi_B}{dt} = +\dfrac{B\ell\, dx}{dt}$

$= B\ell v$

(b) $I = \dfrac{\text{emf}}{R} = \dfrac{B\ell v}{R}$

(c) $P = I^2 R = (B\ell v)^2/R$

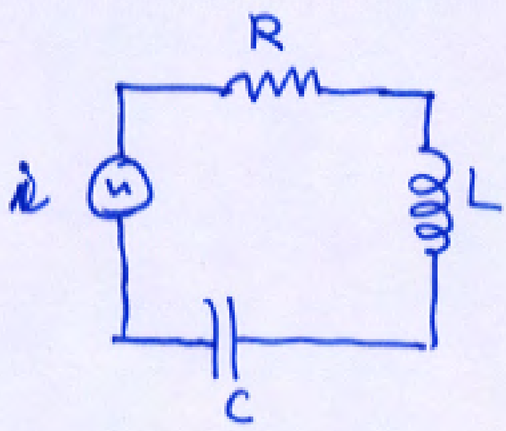
(d) $\vec{F} = I\vec{\ell} \times \vec{B}$

$F = I\ell B = (B\ell)^2 \dfrac{v}{R}$

direction towards left

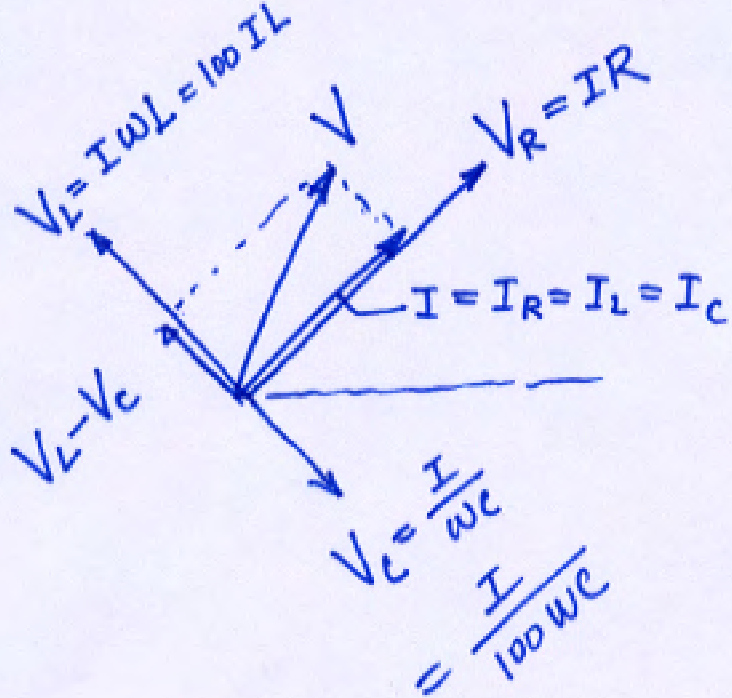
8 (15pt) A series circuit consisting of a current source of i=Isin(100t+45°), an inductor of inductance L, a capacitor of capacitance C, and a resistor of resistance R. Draw a phasor diagram at t=0 containing V_L, V_C, V_R, I_L, I_C, I_R, and the total voltage V. Express these quantities in terms of given values.

5min



In a series circuit, current is same through all element

at t = 0, the current has a phase angle of 45



$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = I\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= I\sqrt{R^2 + \left(100L - \frac{1}{100C}\right)^2}$$